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1 Solve in real numbers $\frac{(x+2)^4}{x^3} - \frac{(x+2)^2}{2x} \geq -\frac{x}{16}$

2 Let ABC be an acute-angled triangle with $AB < AC$. Let D be the midpoint of side BC and BE, CZ be the altitudes of the triangle ABC. Line ZE intersects line BC at point O.
(i) Find all the angles of the triangle ZDE in terms of angle $\angle A$ of the triangle ABC
(ii) Find the angle $\angle BOZ$ in terms of angles $\angle B$ and $\angle C$ of the triangle ABC

3 Find all positive integers x, for which the equation

$$a + b + c = xabc$$

has solution in positive integers.
Solve the equation for these values of x

4 We are having 99 equal circles in a row and in the interior, we write inside them all the numbers from 1 up to 99 (one number in each circle). We color each of the circles with one of the two colors available: red and green.
A coloring is good if it has the ability:
Red circles lying in the interval of the numbers from 1 up to 50 are more than the red circles lying in the interval of the numbers from 51 up to 99.
a) Find how many different colorings can be constructed.
b) Find how many different good colorings can be constructed.
(Note: Two colorings are different, if they have different color in at least one of their circles.)