National Matematical Olympiad 2017
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–	2nd Round
1	The incircle of $\triangle\ ABC$ touches the sides BC, CA, AB at D, E, F respectively. A circle through A and B encloses $\triangle\ ABC$ and intersects the line DE at points P and Q. Prove that the midpoint of AB lies on the circumircle of $\triangle\ PQF$.
2	Let $a_1, a_2, ..., a_n, b_1, b_2, ..., b_n, p$ be real numbers with $p > -1$. Prove that $$\sum_{i=1}^{n}(a_i - b_i)\left(a_i(a_1^2 + a_2^2 + ... + a_n^2)^{p/2} - b_i(b_1^2 + b_2^2 + ... + b_n^2)^{p/2}\right) \geq 0$$
3	Find the smallest positive integer n so that $\sqrt{\frac{1^2+2^2+...+n^2}{n}}$ is an integer.
4	Let $n > 3$ be an integer. Prove that there exist positive integers $x_1, ..., x_n$ in geometric progression and positive integers $y_1, ..., y_n$ in arithmetic progression such that $x_1 < y_1 < x_2 < y_2 < ... < x_n < y_n$
5	Let A and B be two $n \times n$ square arrays. The cells of A are labelled by the numbers from 1 to n^2 from left to right starting from the top row, whereas the cells of B are labelled by the numbers from 1 to n^2 along rising north-easterly diagonals starting with the upper left-hand corner. Stack the array B on top of the array A. If two overlapping cells have the same number, they are coloured red. Determine those n for which there is at least one red cell other than the cells at top left corner, bottom right corner and the centre (when n is odd). Below shows the arrays for $n = 4$. `https://cdn.artofproblemsolving.com/attachments/8/e/cc8a435cb28420ccf91340023d440e39f0e84 png`