National Matematical Olympiad 2016

www.artofproblemsolving.com/community/c1109939

by parmenides51, vlwk

–	2nd Round
1	Let D be a point in the interior of $\triangle ABC$ such that $AB = ab$, $AC = ac$, $BC = bc$, $AD = ad$, $BD = bd$, $CD = cd$. Show that $\angle ABD + \angle ACD = 60^\circ$. Source: 2016 Singapore Mathematical Olympiad (Open) Round 2, Problem 1
2	Let a, b, c be real numbers such that $0 < a, b, c < 1/2$ and $a + b + c = 1$. Prove that for all real numbers x, y, z, $$abc(x + y + z)^2 \geq ayz(1 - 2a) + bxz(1 - 2b) + cxy(1 - 2c)$$. When does equality hold?
3	Let n be a prime number. Show that there is a permutation $a_1, a_2, ..., a_n$ of $1, 2, ..., n$ so that $a_1, a_1a_2, ..., a_1a_2...a_n$ leave distinct remainders when divided by n
4	Let b be a number with $-2 < b < 0$. Prove that there exists a positive integer n such that all the coefficients of the polynomial $(x + 1)^n(x^2 + bx + 1)$ are positive.
5	A total of 731 objects are put into n nonempty bags where n is a positive integer. These bags can be distributed into 17 red boxes and also into 43 blue boxes so that each red and each blue box contain 43 and 17 objects, respectively. Find the minimum value of n.