

**Estonia Team Selection Test 2007**

[www.artofproblemsolving.com/community/c1113557](http://www.artofproblemsolving.com/community/c1113557)

by parmenides51

---

– Day 1

---

**1** On the control board of a nuclear station, there are  $n$  electric switches ( $n > 0$ ), all in one row. Each switch has two possible positions: up and down. The switches are connected to each other in such a way that, whenever a switch moves down from its upper position, its right neighbour (if it exists) automatically changes position. At the beginning, all switches are down. The operator of the board first changes the position of the leftmost switch once, then the position of the second leftmost switch twice etc., until eventually he changes the position of the rightmost switch  $n$  times. How many switches are up after all these operations?

---

**2** Let  $D$  be the foot of the altitude of triangle  $ABC$  drawn from vertex  $A$ . Let  $E$  and  $F$  be points symmetric to  $D$  w.r.t. lines  $AB$  and  $AC$ , respectively. Let  $R_1$  and  $R_2$  be the circumradii of triangles  $BDE$  and  $CDF$ , respectively, and let  $r_1$  and  $r_2$  be the inradii of the same triangles. Prove that  $|S_{ABD} - S_{ACD}| > |R_1r_1 - R_2r_2|$

---

**3** Let  $n$  be a natural number,  $n > 2$ . Prove that if  $\frac{b^n - 1}{b - 1}$  is a prime power for some positive integer  $b$  then  $n$  is prime.

---

– Day 2

---

**4** In square  $ABCD$ , points  $E$  and  $F$  are chosen in the interior of sides  $BC$  and  $CD$ , respectively. The line drawn from  $F$  perpendicular to  $AE$  passes through the intersection point  $G$  of  $AE$  and diagonal  $BD$ . A point  $K$  is chosen on  $FG$  such that  $|AK| = |EF|$ . Find  $\angle EKF$ .

---

**5** Find all continuous functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that for all reals  $x$  and  $y$ ,  $f(x + f(y)) = y + f(x + 1)$ .

---

**6** Consider a  $10 \times 10$  grid. On every move, we colour 4 unit squares that lie in the intersection of some two rows and two columns. A move is allowed if at least one of the 4 squares is previously uncoloured. What is the largest possible number of moves that can be taken to colour the whole grid?

---