National Mathematical Olympiad 1995

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– 2nd Round

1 Suppose that the rational numbers a, b and c are the roots of the equation $x^3 + ax^2 + bx + c = 0$. Find all such rational numbers a, b and c. Justify your answer

2 Let $A_1A_2A_3$ be a triangle and M an interior point. The straight lines MA_1, MA_2, MA_3 intersect the opposite sides at the points B_1, B_2, B_3 respectively (see Fig.). Show that if the areas of triangles A_2B_1M, A_3B_2M and A_1B_3M are equal, then M coincides with the centroid of triangle $A_1A_2A_3$.
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png

3 Let P be a point inside $\triangle\ ABC$. Let D, E, F be the feet of the perpendiculars from P to the lines BC, CA and AB, respectively (see Fig.). Show that
(i) $EF = AP \sin A$,
(ii) $PA + PB + PC \ge 2(PE + PD + PF)$
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4 Let a, b and c be positive integers such that $1 < a < b < c$. Suppose that $(ab - l)(bc - 1)(ca - 1)$ is divisible by abc. Find the values of a, b and c. Justify your answer.

5 Let a, b, c, d be four positive real numbers. Prove that

$$a^{10}+b^{10}+c^{10}+d^{10} \ge (0.1a+0.2b+0.3c+0.4d)^{10}+(0.4a+0.3b+0.2c+0.ld)^{10}+(0.2a+0.4b+0.1c+0.3d)^{10}+(0.3a+$$