AoPS Online
Art of Problem Solving

National Mathematical Olympiad 1997
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by parmenides51

–	2nd Round
1	$\triangle$ ABC is an equilateral triangle. L, M and N are points on BC, CA and AB respectively. Prove that $MA \cdot AN + NB \cdot BL + LC \cdot CM < BC^2$.
2	Observe that the number 4 is such that $\binom{4}{k} = \frac{4!}{k!(4-k)!}$ divisible by $k+1$ for $k = 0, 1, 2, 3$. Find all the natural numbers n between 50 and 90 such that $\binom{n}{k}$ is divisible by $k+1$ for $k = 0, 1, 2, ..., n-1$. Justify your answers.
3	Find all the natural numbers N which satisfy the following properties: (i) N has exactly 6 distinct factors $1, d_1, d_2, d_3, d_4, N$ and (ii) $1 + N = 5(d_1 + d_2 + d_3 + d_4)$. Justify your answers.
4	Let $n \geq 2$ be a positive integer. Suppose that $a_1, a_2, ..., a_n$ and $b_1, b_2, ..., b_n$ are 2n numbers such that $\sum_{i=1}^n a_i = \sum_{i=1}^n n_i = 1$ and $a_i \geq 0, 0 \leq b_i \leq \frac{n-1}{n}, i = 1, 2, ..., n$. Show that $$b_1a_2a_3...a_n + a_1b_2a_3...a_n + ... + a_1a_2...a_{k-1}b_ka_{k+1}...a_n + ... + a_1a_2...a_{n-1}b_n \leq \frac{1}{n(n-1)^{n-2}}$$