National Mathematical Olympiad 1998

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– 2nd Round

1 In Fig. , PA and QB are tangents to the circle at A and B respectively. The line AB is extended to meet PQ at S. Suppose that $PA = QB$. Prove that $QS = SP$.
`https://cdn.artofproblemsolving.com/attachments/6/f/f21c0c70b37768f3e80e9ee909ef34c57635c`
`png`

2 Let N be the set of natural numbers, and let $f : N \to N$ be a function satisfying $f(x)+f(x+2) < 2f(x+1)$ for any $x \in N$. Prove that there exists a straight line in the xy-plane which contains infinitely many points with coordinates $(n, f(n))$.

3 Do there exist integers x and y such that $19^{19} = x^3 + y^4$? Justify your answer.

4 Let n be a fixed positive integer. Find all the positive integers m such that

$$\frac{m^2+4m}{a_1}+\frac{m^2+8m}{a_1+a_2}+\frac{m^2+12m}{a_1+a_2+a_3}+...+\frac{m^2+4nm}{a_1+a_2+...+a_n}<2500\left(\frac{1}{a_1}+\frac{1}{a_2}+...+\frac{1}{a_n}\right)$$

for any positive numbers $a_1, a_2, ..., a_n$. Justify your answer.