National Mathematical Olympiad 1999

www.artofproblemsolving.com/community/c1118755

by parmenides51

– 2nd Round

1 Let n be a positive integer. A square $ABCD$ is divided into n^2 identical small squares by drawing $(n-1)$ equally spaced lines parallel to the side AB and another $(n-1)$ equally spaced lines parallel to BC, thus giving rise to $(n+1)^2$ intersection points. The points A, C are coloured red and the points B, D are coloured blue. The rest of the intersection points are coloured either red or blue. Prove that the number of small squares having exactly 3 vertices of the same colour is even.

2 Call a natural number n a *magic* number if the number obtained by putting n on the right of any natural number is divisible by n. Find the number of magic numbers less than 500. Justify your answer

3 For each positive integer n, let $f(n)$ be a positive integer. Show that if $f(n+1) > f(f(n))$ for every positive integer n, then $f(x) = x$ for all positive integers x.

4 Let $ABCD$ be a quadrilateral with each interior angle less than 180^o. Show that if A, B, C, D do not lie on a circle, then $AB \cdot CD + AD \cdot BC > AC \cdot BD$