National Mathematical Olympiad 2000

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by parmenides51

—   2nd Round

1   Triangle ABC is inscribed in a circle with center O. Let D and E be points on the respective sides AB and AC so that DE is perpendicular to AO. Show that the four points B, D, E and C lie on a circle.

2   Show that 240 divides all numbers of the form $p^4 - q^4$, where p and q are prime numbers strictly greater than 5. Show also that 240 is the greatest common divisor of all numbers of the form $p^4 - q^4$, with p and q prime numbers strictly greater than 5.

3   Is there a positive integer with at most four digits whose value is increased by exactly 60% when the first digit is moved to the end of the number? For example, when the first digit of 1234 is moved to the end of the number, the result is the integer 2341.

4   In a party of 1000 people, the number of people who have shaken hands with at most 962 people is less than or equal to 37. Show that one can find 27 people in the party who have all shaken hands with each other.