National Mathematical Olympiad 2001

www.artofproblemsolving.com/community/c1118788

by parmenides51

–	2nd Round
1	In a parallelogram $ABCD$, the perpendiculars from A to BC and CD meet the line segments BC and CD at the points E and F respectively. Suppose $AC = 37$ cm and $EF = 35$ cm. Let H be the orthocentre of $\triangle\ AEF$. Find the length of AH in cm. Show the steps in your calculations.
2	Let n be a positive integer, and let $a_1, a_2, ..., a_n$ be n positive real numbers such that $a_1 + a_2 + ... + a_n = 1$. Is it true that $\frac{a_1^4}{a_1^2+a_2^2} + \frac{a_2^4}{a_2^2+a_3^2} + \frac{a_3^4}{a_3^2+a_4^2} + ... + \frac{a_{n-1}^4}{a_{n-1}^2+a_n^2} + \frac{a_n^4}{a_n^2+a_1^2} \ge \frac{1}{2n}$? Justify your answer.
3	Suppose that there are 2001 golf balls which are numbered from 1 to 2001 respectively, and some of these golf balls are placed inside a box. It is known that the difference between the two numbers of any two golf balls inside the box is neither 5 nor 8. How many such golf balls the box can contain at most? Justify your answer.
4	A positive integer n is said to possess Property (A) if there exists a positive integer N such that N^2 can be written as the sum of the squares of n consecutive positive integers. Is it true that there are infinitely many positive integers which possess Property (A)? Justify your answer. (As an example, the number $n = 2$ possesses Property (A) since $5^2 = 3^2 + 4^2$).