**National Mathematical Olympiad 2002**

www.artofproblemsolving.com/community/c1118803

by parmenides51

– 2nd Round

1 In the plane, $\Gamma$ is a circle with centre $O$ and radius $r, P$ and $Q$ are distinct points on $\Gamma, A$ is a point outside $\Gamma, M$ and $N$ are the midpoints of $PQ$ and $AO$ respectively. Suppose$OA = 2a$ and $\angle PAQ$ is a right angle. Find the length of $MN$ in terms of $r$ and $a$. Express your answer in its simplest form, and justify your answer.

2 Let $a_1, a_2, ..., a_n$ and $b_1, b_2, ..., b_n$ be real numbers between $1001$ and $2002$ inclusive. Suppose $\sum_{i=1}^n a_i^2 = \sum_{i=1}^n b_i^2$. Prove that

$$\sum_{i=1}^n \frac{a_i^3}{b_i} \le \frac{17}{10} \sum_{i=1}^n a_i^2$$

Determine when equality holds.

3 Let $n$ be a positive integer. Determine the smallest value of the sum $a_1b_1 + a_2b_2 + ... + a_{2n+2}b_{2n+2}$ where $(a_1, a_2, ..., a_{2n+2})$ and $(b_1, b_2, ..., b_{2n+2})$ are rearrangements of the binomial coefficients $\binom{2n+1}{0}$, $\binom{2n+1}{1}$,...,$\binom{2n+1}{2n+1}$. Justify your answer

4 Find all real-valued functions $f : Q \to R$ defined on the set of all rational numbers $Q$ satisfying the conditions $f(x + y) = f(x) + f(y) + 2xy$ for all $x, y$ in $Q$ and $f(1) = 2002$. Justify your answers.