National Mathematical Olympiad 2003

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by parmenides51

–	2nd Round
1	A sequence $(a_1, a_2, ..., a_{675})$ is given so that each term is an alphabet in the English language (no distinction is made between lower and upper case letters). It is known that in the sequence a is never followed by b and c is never followed by d. Show that there are integers m and n with $1 \leq m < n \leq 674$ such that $a_m = a_n$ and $a_{m+1} = a_{n+1}$
2	Find the maximum value of $\frac{xyz}{(1+5x)(4x+3y)(5y+6z)(z+18)}$ as x, y and z range over the set of all positive real numbers. Justify your answer.
3	For any given prime p, determine whether the equation $x^2 + y^2 + p^z = 2003$ always has integer solutions in x, y, z. Justify your answer
4	The pentagon $ABCDE$ which is inscribed in a circle with $AB < DE$ is the base of a pyramid with apex S. If the longest side from S is SA, prove that $BS > CS$.