Thailand Mathematical Olympiad 2011

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– Day 1

1 Given a natural number $n \geq 3$. If p, q are primes, such that, $p \mid n!$ and $q \mid (n-1)! - 1$. Prove that, $p < q$

2 Find all functions $f : \mathbb{N} \to \mathbb{N}$ such that $f(2m+2n) = f(m)f(n)$ for all natural numbers m, n.

3 Given a ΔABC where $\angle C = 90^\circ$, D is a point in the interior of ΔABC and lines AD , BD and CD intersect BC, CA and AB at points P ,Q and R ,respectively. Let M be the midpoint of $\overline{PQ}$. Prove that, if $\angle BRP = \angle PRC$ then $MR = MC$.

4 There are 900 students in an International School. There are 59 international boys and 59 international girls. The Students are partitioned into 30 classrooms (each classrooms have equal number of student) and in each of the classrooms, the student will labelled number from 1 to 30. The Partition must satisfy at least one follow condition:

- Any Two international boys in same classroom can't have consecutive numbers.
- For every classroom, the student who is labelled 1 must be a boy.

Prove that there are 2 classrooms, each of which has 2 international boys with their labels difference equal.

5 Find all n such that

$$n = d(n)^4$$

Where $d(n)$ is the number of divisors of n, for example $n = 2 \cdot 3 \cdot 5 \implies d(n) = 2 \cdot 2 \cdot 2$.

6 For any $0 \leq x_1, x_2, \ldots, x_{2011} \leq 1$, Find the maximum value of

$$\sum_{k=1}^{2011} (x_k - m)^2$$

where m is the arithmetic mean of $x_1, x_2, \ldots, x_{2011}$.

– Day 2

7 Let $a, b, c, d \in \mathbb{R}^+$ and suppose that all roots of the equation

$$x^5 - ax^4 + bx^3 - cx^2 + dx = 1$$

are real. Prove

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\leq\frac{3}{5}$$

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8 Given ΔABC and its centroid G, If line AC is tangent to $\odot(ABG)$. Prove that,

$$AB+BC\leq 2AC$$

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9 Prove that, for all $n\in\mathbb{N}$

$$\frac{1}{1}+\frac{1}{3}+\frac{1}{5}+\ldots+\frac{1}{2n+1}\notin\mathbb{Z}$$

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10 Does there exists a function $f:\mathbb{N}\longrightarrow\mathbb{N}$

$$f\left(m+f(n)\right)=f(m)+f(n)+f(n+1)$$

for all $m,n\in\mathbb{N}$?

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11 In ΔABC, Let the Incircle touch $\overline{BC},\overline{CA},\overline{AB}$ at X,Y,Z. Let I_A,I_B,I_C be A,B,$C-$excenters, respectively. Prove that Incenter of ΔABC, orthocenter of ΔXYZ and centroid of $\Delta I_AI_BI_C$ are collinear.

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12 7662 chairs are placed in a circle around the city of Chiang Mai. They are also marked with a label for either 1st, 2nd, or 3rd grade students, so that there are 2554 chairs labeled with each label. The following situations happen, in order

- 2554 students each from the 1st, 2nd, and 3rd grades are given a ball as follows: 1st grade students receive footballs, 2nd grade students receive basketballs, and 3rd grade students receive volleyballs.
- The students go sit in chairs labeled for their grade
- The students simultaneously send their balls to the student to their left, and this happens some positive number of times.

A labelling of the chairs is called *lin-ping* if it is possible for all 1st, 2nd, and 3rd grade students to now hold volleyballs, footballs, and basketballs respectively. Compute the number of *lin-ping* labellings

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