The problems from the CCA Math Bonanza held on 5/23/2015

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– Individual Round

I1 Michael the Mouse finds a block of cheese in the shape of a regular tetrahedron (a pyramid with equilateral triangles for all faces). He cuts some cheese off each corner with a sharp knife. How many faces does the resulting solid have?

2015 CCA Math Bonanza Individual Round#1

I2 The operation $*$ is defined by the following: $a * b = a! - ab - b$. Compute the value of $5 * 8$.

2015 CCA Math Bonanza Individual Round#2

I3 Mark's teacher is randomly pairing his class of 16 students into groups of 2 for a project. What is the probability that Mark is paired up with his best friend, Mike? (There is only one Mike in the class)

2015 CCA Math Bonanza Individual Round#3

I4 Kevin the Koala eats 1 leaf on the first day of its life, 3 leaves on the second, 5 on the third, and in general eats $2n-1$ leaves on the nth day. What is the smallest positive integer $n > 1$ such that the total number of leaves Kevin has eaten his entire n-day life is a perfect sixth power?

2015 CCA Math Bonanza Individual Round#4

I5 Triangle ABC is equilateral with side length 12. Point D is the midpoint of side $\overline{BC}$. Circles A and D intersect at the midpoints of side AB and AC. Point E lies on segment $\overline{AD}$ and circle E is tangent to circles A and D. Compute the radius of circle E.

2015 CCA Math Bonanza Individual Round#5

I6 How many positive integers less than or equal to 1000 are divisible by 2 and 3 but not by 5?

2015 CCA Math Bonanza Individual Round#6

I7 Harry Potter would like to purchase a new owl which cost him 2 Galleons, a Sickle, and 5 Knuts. There are 23 Knuts in a Sickle and 17 Sickles in a Galleon. He currently has no money, but has many potions, each of which are worth 9 Knuts. How many potions does he have to exhange to buy this new owl?

2015 CCA Math Bonanza Individual Round#7

I8 A rectangle has an area of 16 and a perimeter of 18; determine the length of the diagonal of the rectangle.

2015 CCA Math Bonanza Individual Round#8

I9 There is 1 integer in between 300 and 400 (base 10) inclusive such that its last digit is 7 when written in bases 8, 10, and 12. Find this integer, in base 10.

2015 CCA Math Bonanza Individual Round#9

I10 The fourth-degree equation $x^4 - x - 504 = 0$ has 4 roots r_1, r_2, r_3, r_4. If S_x denotes the value of ${r_1}^4 + {r_2}^4 + {r_3}^4 + {r_4}^4$, compute S_4.

2015 CCA Math Bonanza Individual Round#10

I11 A dog owns 4 different color shoes and 4 identical green socks. He can fit every shoe and sock on each of his four distinguishable paws. In how many different orders can he put on the shoes and socks, provided that on each paw he must put on the sock before the shoe?

2015 CCA Math Bonanza Individual Round#11

I12 Positive integers x, y, z satisfy $x^3 + xy + x^2 + xz + y + z = 301$. Compute $y + z - x$.

2015 CCA Math Bonanza Individual Round#12

I13 Let $ABCD$ be a tetrahedron such that $AD \perp BD$, $BD \perp CD$, $CD \perp AD$ and $AD = 10$, $BD = 15$, $CD = 20$. Let E and F be points such that E lies on BC, $DE \perp BC$, and $ADEF$ is a rectangle. If S is the solid consisting of all points inside $ABCD$ but outside $FBCD$, compute the volume of S.

2015 CCA Math Bonanza Individual Round#13

I14 10 children each have a lunchbox which they store in a basket before entering their classroom. However, being messy children, their lunchboxes get mixed up. When leaving the classroom each student picks up a lunchbox at random. Define a *cyclic triple* of students (A, B, C) to be three distinct students such that A has B's lunchbox, B has C's lunchbox, and C has A's lunchbox. Two cyclic triples are considered the same if they contain the same three students (even if in a different order). Determine the expected value of the number of cyclic triples.

2015 CCA Math Bonanza Individual Round#14

I15 Let ω_1 and ω_2 be circles with radii 3 and 12 and externally tangent at point P. Let a common external tangent intersect ω_1, ω_2 at S, T respectively and the common internal tangent at point Q. Define X to be the point on $\overrightarrow{QP}$ such that $QX = 10$. If XS, XT intersect ω_1, ω_2 a second time at A and B, determine $\tan \angle APB$.

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2015 CCA Math Bonanza Individual Round#15

– Team Round

T1 An archery target can be represented as three concentric circles with radii 3, 2, and 1 which split the target into 3 regions, as shown in the figure below. What is the area of Region 1 plus the area of Region 3?

2015 CCA Math Bonanza Team Round#1

T2 How many ways are there to rearrange the letters of the word RAVEN such that no two vowels are consecutive?

2015 CCA Math Bonanza Team Round#2

T3 A tortoise is given an 80-second head start in a race. When Achilles catches up to where the tortoise was when he (Achilles) began running, he finds that while he is now 40 meters ahead of the starting line, the tortoise is now 5 meters ahead of him. At this point, how long will it be, in seconds, before Achilles passes the tortoise?

2015 CCA Math Bonanza Team Round#3

T4 Evaluate the continued fraction

$$1+\frac{2}{2+\frac{2}{2+\ldots}}$$

2015 CCA Math Bonanza Team Round#4

T5 Emily Thorne is throwing a Memorial Day Party to kick off the Summer in the Hamptons, and she is trying to figure out the seating arrangment for all of her guests. Emily saw that if she seated 4 guests to a table, there would be 1 guest left over (how sad); if she seated 5 to a table, there would be 3 guests left over; and if she seated 6 to a table, there would again be 1 guest left over. If there are at least 100 but no more than 200 guests (because she's rich and her house is 20000 square feet), what is the greatest possible number of guests?

2015 CCA Math Bonanza Team Round#5

T6 A semicircle is inscribed in right triangle ABC with right angle B and has diameter on AB, with one end on point B. Given that $AB = 15$ and $BC = 8$, determine the radius of the semicircle

2015 CCA Math Bonanza Team Round#6

T7 At a party, five of Ryan's friends arrive, each hanging their coats on the coat rack. When they leave, Ryan hands out coats in a random order to his friends. What is the probability that at least half of them receive the right coat? (Half of them is 3 or more)

2015 CCA Math Bonanza Team Round#7

T8 Triangle ABC is equilateral with side length $\sqrt{3}$ and circumcenter at O. Point P is in the plane such that $(AP)(BP)(CP) = 7$. Compute the difference between the maximum and minimum possible values of OP.

2015 CCA Math Bonanza Team Round#8

T9 The Fibonacci numbers are defined as the sequence F_n with $F_0 = 1$, $F_1 = 1$ and $F_{n+2} = F_{n+1} + F_n$. How many ways can 10 be written as an ordered sum of numbers found in the Fibonacci sequence? For example, 3 can be written as $1+1+1$, $2+1$, $1+2$, and 3, for a total of 4 ways.

2015 CCA Math Bonanza Team Round#9

T10 If $\cos 2^\circ - \sin 4^\circ - \cos 6^\circ + \sin 8^\circ \ldots + \sin 88^\circ = \sec\theta - \tan\theta$, compute θ in degrees.

2015 CCA Math Bonanza Team Round#10

– Lightning Round

L1.1 What is the value of $(2^{-1})^{-2}$?

2015 CCA Math Bonanza Lightning Round#1.1

L1.2 Let $ABCDEF$ be a regular hexagon with side length 2. Calculate the area of $ABDE$.

2015 CCA Math Bonanza Lightning Round#1.2

L1.3 Daniel can hack a finite cylindrical log into 3 pieces in 6 minutes. How long would it take him to cut it into 9 pieces, assuming each cut takes Daniel the same amount of time?

2015 CCA Math Bonanza Lightning Round#1.3

L1.4 How many digits are in the base 10 representation of 3^{30} given $\log 3 = 0.47712$?

2015 CCA Math Bonanza Lightning Round#1.4

L2.1 What is the sum of the first 10 primes?

2015 CCA Math Bonanza Lightning Round#2.1

L2.2 Find all real x that satisfy the equation

$$\frac{1}{x+1} + \frac{1}{x+2} = \frac{1}{x}$$

2015 CCA Math Bonanza Lightning Round#2.2

L2.3 Find the last digit of the number
$$\frac{400!}{(200!)(2^{200})}$$

2015 CCA Math Bonanza Lightning Round#2.3

L2.4 The polynomial $x^3 - kx^2 + 20x - 15$ has 3 roots, one of which is known to be 3. Compute the greatest possible sum of the other two roots.

2015 CCA Math Bonanza Lightning Round#2.4

L3.1 Bhairav the Bat lives next to a town where 12.5

2015 CCA Math Bonanza Lightning Round#3.1

L3.2 In triangle ABC, points M, N, and P lie on sides $\overline{AC}$, $\overline{AB}$, and $\overline{BC}$, respectively. If $\angle ABC = 42^\circ$, $\angle MAN = 91^\circ$, and $\angle NMA = 47^\circ$, compute $\frac{CB}{BP}$.

2015 CCA Math Bonanza Lightning Round#3.2

L3.3 Michael the Mouse stands in a circle with 11 other mice. Eshaan the Elephant walks around the circle, squashing every other non-squashed mouse he comes across. If it takes Eshaan 1 minute (60 seconds) to complete one circle and he walks at a constant rate, what is the maximum length of time in seconds from when the first mouse is squashed that Michael can survive?

2015 CCA Math Bonanza Lightning Round#3.3

L3.4 Compute the greatest constant K such that for all positive real numbers a, b, c, d measuring the sides of a cyclic quadrilateral, we have
$$\left(\frac{1}{a+b+c-d}+\frac{1}{a+b-c+d}+\frac{1}{a-b+c+d}+\frac{1}{-a+b+c+d}\right)(a+b+c+d) \geq K.$$

2015 CCA Math Bonanza Lightning Round#3.4

L4.1 How many divisors of $12!$ are perfect squares?

2015 CCA Math Bonanza Lightning Round#4.1

L4.2 Let $ABCD$ be a square of side length 1, and let E and F be points on BC and DC such that $\angle EAF = 30^\circ$ and $CE = CF$. Determine the length of BD.

2015 CCA Math Bonanza Lightning Round#4.2

L4.3 Andrew the ant starts at vertex A of square $ABCD$. Each time he moves, he chooses the clockwise vertex with probability $\frac{2}{3}$ and the counter-clockwise vertex with probability $\frac{1}{3}$. What is the probability that he ends up on vertex A after 6 moves?

2015 CCA Math Bonanza Lightning Round#4.3

L4.4 Sierpinski's triangle is formed by taking a triangle, and drawing an upside down triangle inside each upright triangle that appears. A snake sees the fractal, but decides that the triangles need circles inside them. Therefore, she draws a circle inscribed in every upside down triangle she sees (assume that the snake can do an infinite amount of work). If the original triangle had side length 1, what is the total area of all the individual circles?

2015 CCA Math Bonanza Lightning Round#4.4

L5.1 What is the integer closest to π^π? (No calculator allowed!)

2015 CCA Math Bonanza Lightning Round#5.1

L5.2 If a train carrying 27 passengers leaves Grand Central Station at $8:00$ AM and travels 900 miles due west to Chicago, arriving at $5:00$ PM, what is the average speed of the train in miles per hour?

2015 CCA Math Bonanza Lightning Round#5.2

L5.3 Alice the ant starts at vertex A of regular hexagon $ABCDEF$ and moves either right or left each move with equal probability. After 35 moves, what is the probability that she is on either vertex A or C?

2015 CCA Math Bonanza Lightning Round#5.3

L5.4 Submit a positive integer x between 1 and 10 inclusive. Your score on the problem will be proportional to

$$\frac{11-x}{n}$$

where n is the number of teams that also submit the number x.

2015 CCA Math Bonanza Lightning Round#5.4

– Tiebreaker Round

TB1 Compute the greatest 4-digit number $\underline{ABCD}$ such that $(A^3+B^2)(C^3+D^2)=2015$.

2015 CCA Math Bonanza Tiebreaker Round#1

TB2 If a, b, c are the roots of x^3+20x^2+1x+5, compute $(a^2+1)(b^2+1)(c^2+1)$.

2015 CCA Math Bonanza Tiebreaker Round#2

TB3 Positive integers (not necessarily unique) are written, one on each face, on two cubes such that when the two cubes are rolled, each integer $2 \le k \le 12$ appears as the sum of the upper faces with probability $\frac{6-|7-k|}{36}$. Compute the greatest possible sum of all the faces on one cube.

2015 CCA Math Bonanza Tiebreaker Round#3