Czech And Slovak Mathematical Olympiad, Round III, Category A 1983

www.artofproblemsolving.com/community/c1125225

by byk7

---

1 Let n be a positive integer and $k \in [0,n]$ be a fixed real constant. Find the maximum value of

$$\left|\sum_{i=1}^{n}\sin(2x_i)\right|$$

where $x_1,\dots,x_n$ are real numbers satisfying

$$\sum_{i=1}^{n}\sin^2(x_i)=k.$$

---

2 Given a triangle ABC, prove that for every inner point P of the side AB the inequality

$$PC\cdot AB<PA\cdot BC+PB\cdot AC$$

holds.

---

3 An 8×8 chessboard is made of unit squares. We put a rectangular piece of paper with sides of length 1 and 2. We say that the paper and a single square overlap if they share an inner point. Determine the maximum number of black squares that can overlap the paper.

---

4 Consider an arithmetic progression $a_0,\dots,a_n$ with $n\geq 2$. Prove that

$$\sum_{k=0}^{n}(-1)^k\binom{n}{k}a_k=0.$$

---

5 Find all pair (x,y) of positive integers satisfying

$$\left|\frac{x}{y}-\sqrt{2}\right|<\frac{1}{y^3}.$$

---

6 Consider a circle k with center S and radius r. Denote M the set of all triangles with incircle k such that the largest inner angle is twice bigger than the smallest one. For a triangle $\mathcal{T}\in$ M denote its vertices A,B,C in way that $SA\geq SB\geq SC$. Find the locus of points $\{B \mid \mathcal{T}\in \text{M}\}$.
