

Olympic Revenge 2005

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- 1 Let $S = \{1, 2, 3, \dots, n\}$, n an odd number. Find the parity of number of permutations $\sigma : S \Rightarrow S$ such that the sequence defined by

$$a(i) = |\sigma(i) - i|$$

is monotonous.

- 2 Let Γ be a circumference, and A, B, C, D points of Γ (in this order).
 r is the tangent to Γ at point A. s is the tangent to Γ at point D.

Let $E = r \cap BC, F = s \cap BC$.

Let $X = r \cap s, Y = AF \cap DE, Z = AB \cap CD$

Show that the points X, Y, Z are collinear.

Note: assume the existence of all above points.

- 3 Find all functions $f : R \rightarrow R$ such that

$$f(x + yf(x)) + f(xf(y) - y) = f(x) - f(y) + 2xy$$

for all $x, y \in R$

- 4 Let A be a symmetric matrix such that the sum of elements of any row is zero.
Show that all elements in the main diagonal of cofactor matrix of A are equal.

- 5 Find all sets X of points in a plane, not all collinear, such that:
For any two distinct circumferences, each contains three points of X, its intersection points are points of X.

- 6 Z Roberto and Humberto are playing the Millenium Game!
There are 30 empty boxes in a queue, and each box have a capacity of one blue stome.
Each player takes a blue stone and places it in a box (and it is a *move*).
The winner is who, in its move, obtain three full consecutive boxes.

If Z Roberto is the first player, who has the winner strategy?