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– Day 1

1 Given an integer $n \geq 2$ and a regular $2n$-polygon at each vertex of which sitting on an ant. At some points in time, each ant creeps into one of two adjacent peaks (some peaks may have several ants at a time). Through k such operations, it turned out to be an arbitrary line connecting two different ones the vertices of a polygon with ants do not pass through its center. For given n find the lowest possible value of k.

2 Let $x_1, x_2, \cdots, x_n$ be postive real numbers such that $x_1x_2\cdots x_n = 1$,$S = x_1^3 + x_2^3 + \cdots + x_n^3$.Find the maximum of $\frac{x_1}{S-x_1^3+x_1^2} + \frac{x_2}{S-x_2^3+x_2^2} + \cdots + \frac{x_n}{S-x_n^3+x_n^2}$

3 Let $ABCDEF$ be a convex hexagon with $AB = DE$, $BC = EF$, $CD = FA$, and $\angle A - \angle D = \angle C - \angle F = \angle E - \angle B$. Prove that the diagonals AD, BE, and CF are concurrent.

– Day 2

4 The A-excircle of the triangle ABC touches the side BC at point K. The circumcircles of triangles AKB and AKC intersect for the second time with the bisector of angle A at points X and Y respectively. Let M be the midpoint of BC. Prove that the circumcenter of triangle XYM lies on BC.

5 Find all positive integers $n \geq 2$ such that equality $i + j \equiv C_n^i + C_n^j$ (mod 2) is true for arbitrary $0 \leq i \leq j \leq n$.

6 Let $n \geq 3$ be an odd integer. Each cell is a $n \times n$ board painted in yellow or blue. Let's call the sequence of cells $S_1, S_2, ..., S_m$ *path* if they are all the same color and the cells S_i and S_j have one in common an edge if and only if $|i - j| = 1$. Suppose that all yellow cells form a path and all the blue cells form a path. Prove that one of the two paths begins or ends at the center of the board.

– Day 3

7 For each natural $n \geq 4$, find the smallest natural number k that satisfies following condition: For an arbitrary arrangement of k chips of two colors on $n \times n$ board, there exists a non-empty set such that all columns and rows contain even number (0 is also possible) of chips each color.

8 The quadrilateral $ABCD$ is inscribed in the circle ω with the center O. Suppose that the angles B and C are obtuse and lines AD and BC are not parallel. Lines AB and CD intersect at point E. Let P and R be the feet of the perpendiculars from the point E on the lines BC and AD respectively. Q is the intersection point of EP and AD, S is the intersection point of ER and BC. Let K be the midpoint of the segment QS . Prove that the points E, K, and O are collinear.

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9 Let m, n be odd prime numbers.
Find all pairs of integers numbers a, b for which the system of equations: $x^m + y^m + z^m = a$, $x^n + y^n + z^n = b$
has many solutions in integers x, y, z.

– Day 4

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10 Find all positive integers $n \geq 4$ for which there are n points in general position on the plane such that an arbitrary triangle with vertices belonging to the convex hull of these n points, containing exactly one of $n-3$ points inside remained.

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11 Find all functions $f : R \to R$ that satisfy the condition $(f(x)-f(y))(u-v) = (f(u)-f(v))(x-y)$ for arbitrary real x, y, u, v such that $x + y = u + v$.

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12 Prove that for an arbitrary prime $p \geq 3$ the number of positive integers n, for which $p|n! + 1$ does not exceed $cp^{2/3}$, where c is a constant that does not depend on p.
