Nordic 2020

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by parmenides51

1 For a positive integer n, denote by $g(n)$ the number of strictly ascending triples chosen from the set $\{1, 2, ..., n\}$. Find the least positive integer n such that the following holds:[i] The number $g(n)$ can be written as the product of three different prime numbers which are (not necessarily consecutive) members in an arithmetic progression with common difference 336.[/i]

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2 Georg has $2n + 1$ cards with one number written on each card. On one card the integer 0 is written, and among the rest of the cards, the integers $k = 1, ..., n$ appear, each twice. Georg wants to place the cards in a row in such a way that the 0-card is in the middle, and for each $k = 1, ..., n$, the two cards with the number k have the distance k (meaning that there are exactly $k - 1$ cards between them).

For which $1 \leq n \leq 10$ is this possible?

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3 Each of the sides AB and CD of a convex quadrilateral $ABCD$ is divided into three equal parts, $|AE| = |EF| = |FB|$, $|DP| = |PQ| = |QC|$. The diagonals of $AEPD$ and $FBCQ$ intersect at M and N, respectively. Prove that the sum of the areas of $\triangle AMD$ and $\triangle BNC$ is equal to the sum of the areas of $\triangle EPM$ and $\triangle FNQ$.

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4 Find all functions $f : R - \{-1\} \to R$ such that

$$f(x)f\left(f\left(\frac{1-y}{1+y}\right)\right) = f\left(\frac{x+y}{xy+1}\right)$$

for all $x, y \in R$ that satisfy $(x + 1)(y + 1)(xy + 1) \neq 0$.
