Niels Henrik Abels Math Contest (Norwegian Math Olympiad) Final Round 2020

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by parmenides51

1a In how many ways can the circles be coloured using three colours, so that no two circles connected by a line segment have the same colour?
https://cdn.artofproblemsolving.com/attachments/3/2/e2bd61786aa4269593233311e85204cff071e
png

1b A round table has room for n diners ($n \geq 2$). There are napkins in three different colours. In how many ways can the napkins be placed, one for each seat, so that no two neighbours get napkins of the same colour?

2a Find all natural numbers k such that there exist natural numbers $a_1, a_2, ..., a_{k+1}$ with $a_1! + a_2! + ... + a_{k+1}! = k!$
Note that we do not consider 0 to be a natural number.

2b Assume that a and b are natural numbers with $a \geq b$ so that $\sqrt{a + \sqrt{a^2 - b^2}}$ is a natural number. Show that a and b have the same parity.

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3 Show that the equation $x^2 \cdot (x-1)^2 \cdot (x-2)^2 \cdot ... \cdot (x-1008)^2 \cdot (x-1009)^2 = c$ has 2020 real solutions, provided $0 < c < \frac{(1009 \cdot 1007 \cdot ... \cdot 3 \cdot 1)^4}{2^{2020}}$.

4a The midpoint of the side AB in the triangle ABC is called C'. A point on the side BC is called D, and E is the point of intersection of AD and CC'. Assume that $AE/ED = 2$. Show that D is the midpoint of BC.

4b The triangle ABC has a right angle at A. The centre of the circumcircle is called O, and the base point of the normal from O to AC is called D. The point E lies on AO with $AE = AD$. The angle bisector of $\angle CAO$ meets CE in Q. The lines BE and OQ intersect in F. Show that the lines CF and OE are parallel.
