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by parmenides51

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| – | Day 1 |
| 1 | Let $a, b, c$ be positive reals. Prove that $\sqrt{2a^2+bc}+\sqrt{2b^2+ac}+\sqrt{2c^2+ab} \ge 3\sqrt{ab+bc+ca}$ |
| 2 | $E$ is the intersection point of the diagonals of the cyclic quadrilateral, $ABCD, F$ is the intersection point of the lines $AB$ and $CD$, $M$ is the midpoint of the side $AB$, and $N$ is the midpoint of the side $CD$. The circles circumscribed around the triangles $ABE$ and $ACN$ intersect for the second time at point $K$. Prove that the points $F, K, M$ and $N$ lie on one circle. |
| 3 | A natural number $n$ is called *perfect* if it is equal to the sum of all its natural divisors other than $n$. For example, the number $6$ is perfect because $6 = 1 + 2 + 3$. Find all even perfect numbers that can be given as the sum of two cubes positive integers. |
| – | Day 2 |
| 4 | Given an isosceles triangle $ABC$ ($AB = AC$), the inscribed circle $\omega$ touches its sides $AB$ and $AC$ at points $K$ and $L$, respectively. On the extension of the side of the base $BC$, towards $B$, an arbitrary point $M$. is chosen. Line $M$ intersects $\omega$ at the point $N$ for the second time, line $BN$ intersects the second point $\omega$ at the point $P$. On the line $PK$, there is a point $X$ such that $K$ lies between $P$ and $X$ and $KX = KM$. Determine the locus of the point $X$. |
| 5 | There are only two letters in the Mumu tribe alphabet: M and $U$. The word in the Mumu language is any sequence of letters $M$ and $U$, in which next to each letter $M$ there is a letter $U$ (for example, $UUU$ and $UMMUM$ are words and $MMU$ is not). Let $f(m, u)$ denote the number of words in the Mumu language which have $m$ times the letter $M$ and $u$ times the letter $U$. Prove that $f(m,u) - f(2u-m+1,u) = f(m,u-1) - f(2u-m+1,u-1)$ for any $u \ge 2, 3 \le m \le 2u$. |
| 6 | For the positive integer $k$ we denote by the $a_n$ , the $k$ from the left digit in the decimal notation of the number $2^n$ ($a_n = 0$ if in the notation of the number $2^n$ less than the digits). Consider the infinite decimal fraction $a = \overline{0,a_1a_2a_3...}$. Prove that the number $a$ is irrational. |
| – | Day 3 |
| 7 | Find all pairs of relatively prime integers $(x, y)$ that satisfy equality $2(x^3-x) = 5(y^3-y)$. |
| 8 | Call arrangement of $m$ number on the circle [b]$m$-negative[/b], if all numbers are equal to $-1$. On the first step Andrew chooses one number on circle and multiplies it by $-1$. All other steps are |

similar: instead of the next number(clockwise) he writes its product with the number, written on the previous step. Prove that if $n$-negative arrangement in $k$ steps becomes $n$-negative again, then $(2^n-1)$-negative after $(2^k-1)$ steps becomes $(2^n-1)$-negative again.

**9** The inscribed circle $\omega$ of the triangle $ABC$ touches its sides $BC, CA$ and $AB$ at points $A_1, B_1$ and $C_1$, respectively. Let $S$ be the intersection point of lines passing through points $B$ and $C$ and parallel to $A_1C_1$ and $A_1B_1$ respectively, $A_0$ be the foot of the perpendicular drawn from point $A_1$ on $B_1C_1$, $G_1$ be the centroid of triangle $A_1B_1C_1$, $P$ be the intersection point of the ray $G_1A_0$ with $\omega$. Prove that points $S$, $A_1$, and $P$ lie on a straight line.

**–** Day 4

**10** A unit square is cut by $n$ straight lines . Prove that in at least one of these parts one can completely fit a square with side $\frac{1}{n+1}$

$n$ . , $\frac{1}{n+1}$

The selection panel jury made a mistake because the solution known to it turned out to be incorrect. As it turned out, the assertion of the problem is still correct, although it cannot be proved by simple methods, see. article:
Keith Ball. he plank problem for symmetric bodies // . 1991. . 104, . 1. . 535-543. `https://arxiv.org/abs/math/9201218`

**11** Let $P$ be a polynomial with integer coefficients of degree $d$. For the set $A=\{a_1, a_2, ..., a_k\}$ of positive integers we denote $S(A)=P(a_1)+P(a_2)+...+P(a_k)$. The natural numbers $m, n$ are such that $m^{d+1}|n$. Prove that the set $\{1, 2, ..., n\}$ can be subdivided into $m$ disjoint subsets $A_1, A_2, ..., A_m$ with the same number of elements such that $S(A_1)=S(A_2)=...=S(A_m)$.

**12** We shall call the triplet of numbers $a, b, c$ of the interval $[-1, 1]$ *qualitative* if these numbers satisfy the inequality $1+2abc \geq a^2+b^2+c^2$. Prove that when the triples $a, b, c$, and $x, y, z$ are qualitative, then $ax, by, cz$ is also qualitative.