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1 Find all positive integers d with the following property: there exists a polynomial P of degree d with integer coefficients such that $|P(m)| = 1$ for at least $d+1$ different integers m.

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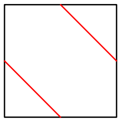
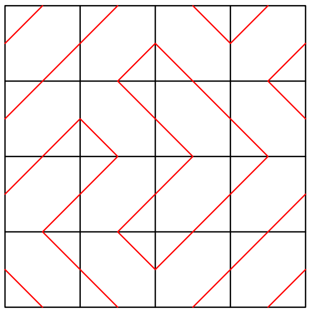
2 Let N be a positive integer. A collection of $4N^2$ unit tiles with two segments drawn on them as shown is assembled into a $2N \times 2N$ board. Tiles can be rotated.



The segments on the tiles define paths on the board. Determine the least possible number and the largest possible number of such paths.

[i]For example, there are 9 paths on the 4×4 board shown below.[/i]



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3 Let ABC be a triangle. The circle ω_A through A is tangent to line BC at B. The circle ω_C through C is tangent to line AB at B. Let ω_A and ω_C meet again at D. Let M be the midpoint of line segment $[BC]$, and let E be the intersection of lines MD and AC. Show that E lies on ω_A.

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4 A divisor d of a positive integer n is said to be a *close* divisor of n if $\sqrt{n} < d < 2\sqrt{n}$. Does there exist a positive integer with exactly 2020 close divisors?
