Argentina National Olympiad 2008

www.artofproblemsolving.com/community/c1142401

by parmenides51, uglysolutions

–	Level 3
–	Day 1
1	101 positive integers are written on a line. Prove that we can write signs $+$, signs $\times$ and parenthesis between them, without changing the order of the numbers, in such a way that the resulting expression makes sense and the result is divisible by $16!$.
2	In every cell of a 60×60 board is written a real number, whose absolute value is less or equal than 1. The sum of all numbers on the board equals 600. Prove that there is a 12×12 square in the board such that the absolute value of the sum of all numbers on it is less or equal than 24.
3	On a circle of center O, let A and B be points on the circle such that $\angle AOB = 120^o$. Point C lies on the small arc AB and point D lies on the segment AB. Let also $AD = 2, BD = 1$ and $CD = \sqrt{2}$. Calculate the area of triangle ABC.
–	Day 2
4	Find all real numbers x which satisfy the following equation: $[2x] + [3x] + [7x] = 2008$. Note: $[x]$ means the greatest integer less or equal than x.
5	Find all perfect powers whose last 4 digits are $2, 0, 0, 8$, in that order.
6	Consider a board of $a \times b$, with a and b integers greater than or equal to 2. Initially their squares are colored black and white like a chess board. The permitted operation consists of choosing two squares with a common side and recoloring them as follows: a white square becomes black; a black box turns green; a green box turns white. Determine for which values of a and b it is possible, by a succession of allowed operations, to make all the squares that were initially white end black and all the squares that were initially black end white. Clarification: Initially there are no green squares, but they appear after the first operation.