www.artofproblemsolving.com/community/c116578
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1 Let $V = [(x,y,z)|0 \le x,y,z \le 2008]$ be a set of points in a 3-D space.
If the distance between two points is either $1, \sqrt{2}, 2$, we color the two points differently.
How many colors are needed to color all points in V?

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2 We have $x_i > i$ for all $1 \le i \le n$.
Find the minimum value of $\frac{(\sum_{i=1}^n x_i)^2}{\sum_{i=1}^n \sqrt{x_i^2 - i^2}}$

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3 Points A, B, C, D, E lie in a counterclockwise order on a circle O, and $AC = CE$ $P = BD \cap AC$, $Q = BD \cap CE$
Let O_1 be the circle which is tangent to $\overline{AP}, \overline{BP}$ and arc AB (which doesn't contain C)
Let O_2 be the circle which is tangent $\overline{DQ}, \overline{EQ}$ and arc DE (which doesn't contain C)
Let $O_1 \cap O = R, O_2 \cap O = S, RP \cap QS = X$
Prove that XC bisects $\angle ACE$

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4 We define A, B, C as a *partition* of $\mathbb{N}$ if A, B, C satisfies the following.
(i) $A, B, C \ne \phi$ (ii) $A \cap B = B \cap C = C \cap A = \phi$ (iii) $A \cup B \cup C = \mathbb{N}$.

Prove that the partition of $\mathbb{N}$ satisfying the following does not exist.
(i) $\forall\ a \in A, b \in B$, we have $a + b + 2008 \in C$
(ii) $\forall\ b \in B, c \in C$, we have $b + c + 2008 \in A$
(iii) $\forall\ c \in C, a \in A$, we have $c + a + 2008 \in B$

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5 Let p be a prime where $p \ge 5$.
Prove that $\exists n$ such that $1 + (\sum_{i=2}^n \frac{1}{i^2})(\prod_{i=2}^n i^2) \equiv 0 \pmod p$

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6 Let $ABCD$ be inscribed in a circle ω.
Let the line parallel to the tangent to ω at A and passing D meet ω at E. F is a point on ω such that lies on the different side of E wrt CD.
If $AE \cdot AD \cdot CF = BE \cdot BC \cdot DF$ and $\angle CFD = 2\angle AFB$,
Show that the tangent to ω at A, B and line EF concur at one point.
(A and E lies on the same side of CD)

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7 Prove that the only function $f : \mathbb{R} \to \mathbb{R}$ satisfying the following is $f(x) = x$.
(i) $\forall x \ne 0$, $f(x) = x^2 f(\frac{1}{x})$.
(ii) $\forall x, y$, $f(x+y) = f(x) + f(y)$.
(iii) $f(1) = 1$.

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8 For fixed positive integers s, t, define a_n as the following. $a_1 = s, a_2 = t$, and $\forall n \geq 1$, $a_{n+2} = \lfloor \sqrt{a_n + (n+2)a_{n+1} + 2008} \rfloor$.
Prove that the solution set of $a_n \neq n$, $n \in \mathbb{N}$ is finite.