239 Open Mathematical Olympiad 2014

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–	Grade 10-11
1	Two players take turns alternatively and remove a number from $1, 2, \dots, 1000$. Players can not remove a number that differ with a number already removed by 1 also they can not remove a number such that it sums up with another removed number to 1001. The player who can not move loses. Determine the winner.
2	The fourth-degree polynomial $P(x)$ is such that the equation $P(x) = x$ has 4 roots, and any equation of the form $P(x) = c$ has no more two roots. Prove that the equation $P(x) = -x$ too has no more than two roots.
3	A natural number is called *good* if it can be represented as sum of two coprime natural numbers, the first of which decomposes into odd number of primes (not necceserily distinct) and the second to even. Prove that there exist infinity many n with n^4 being good.
4	The median CM of the triangle ABC is equal to the bisector BL, also $\angle BAC = 2\angle ACM$. prove that the triangle is right.
5	Find all possible values of k such that there exist a $k \times k$ table colored in k colors such that there do not exist two cells in a column or a row with the same color or four cells made of intersecting two columns and two rows painted in exactly three colors.
6	Given posetive real numbers $a_1, a_2, \dots, a_n$ such that $a_1^2 + 2a_2^3 + \dots + na_n^{n+1} < 1$. Prove that $2a_1 + 3a_2^2 + \dots + (n+1)a_n^n < 3$.
7	A circle ω is strictly inside triangle ABC. The tangents from A to ω intersect BC in A_1, A_2 define B_1, B_2, C_1, C_2 similarly. Prove that if five of six points $A_1, A_2, B_1, B_2, C_1, C_2$ lie on a circle the sixth one lie on the circle too.
8	Prove that the for all $n > 1000$, we can arrange the number $1, 2, \dots, \binom{n}{2}$ on edges of a complete graph with n vertices so that the sum of the numbers assigned to edges of any length three path (possibly closed) is not less than $3n - 1000log_2log_2n$.