239 Open Mathematical Olympiad 2011

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–	Grade 10-11
1	Positive integers a, b, c satisfy that $a+b=b(a-c)$ and c+1 is a square of a prime. Prove that $a+b$ or ab is a square.
2	There are 100 people in the group. Is it possible that for each pair of people exist at least 50 others, so every in that group knows exactly one person from the pair?
3	Positive reals a, b, c, d satisfy $a+b+c+d=4$. Prove that $\sum_{cyc} \frac{a}{a^3+4} \leq \frac{4}{5}$
4	In convex quadrilateral $ABCD$, where $AB=AD$, on BD point K is chosen. On KC point L is such that $\triangle BAD \sim \triangle BKL$. Line parallel to DL and passes through K, intersect CD at P. Prove that $\angle APK = \angle LBC$. @below edited
5	Prove that there exist 1000 consecutive numbers such that none of them is divisible by its sum of the digits
6	Some regular polygons are inscribed in a circle. Fedir turned some of them, so all polygons have a common vertice. Prove that the number of vertices did not increase.
–	Grade 8-9
1	In the acute triangle ABC on AC point P is chosen such that $2AP=BC$. Points X and Y are symmetric to P wrt A and C respectively. It turned out that $BX=BY$. Find angle C.
2	Same as grade 10-11, 1
3	Same as grade 10-11, 2
4	Rombus ABCD with acute angle B is given. O is a circumcenter of ABC. Point P lies on line OC beyond C. PD intersect the line that goes through O and parallel to AB at Q. Prove that $\angle AQO = \angle PBC$.
5	There are 20 blue points on the circle and some red inside so no three are collinear. It turned out that there exists 1123 triangles with blue vertices having 10 red points inside. Prove that all triangles have 10 red points inside

6	Same as grade 10-11, 5
7	Prove for positive reals a, b, c that $(ab+bc+ca+1)(a+b)(b+c)(c+a) \geq 2abc(a+b+c+1)^2$