Greece Team Selection Test 2019

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1 Given an equilateral triangle with sidelength k cm. With lines parallel to it's sides, we split it into k^2 small equilateral triangles with sidelength 1 cm. This way, a triangular grid is created. In every small triangle of sidelength 1 cm, we place exactly one integer from 1 to k^2 (included), such that there are no such triangles having the same numbers. With vertices the points of the grid, regular hexagons are defined of sidelengths 1 cm. We shall name as *value* of the hexagon, the sum of the numbers that lie on the 6 small equilateral triangles that the hexagon consists of . Find (in terms of the integer $k > 4$) the maximum and the minimum value of the sum of the values of all hexagons .

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2 Let a triangle ABC inscribed in a circle Γ with center O. Let I the incenter of triangle ABC and D, E, F the contact points of the incircle with sides BC, AC, AB of triangle ABC respectively . Let also S the foot of the perpendicular line from D to the line EF.Prove that line SI passes from the antidiametric point N of A in the circle Γ.(AN is a diametre of the circle Γ).

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3 Let $n > 1$ be a positive integer. Each cell of an $n \times n$ table contains an integer. Suppose that the following conditions are satisfied:

- Each number in the table is congruent to 1 modulo n.
- The sum of numbers in any row, as well as the sum of numbers in any column, is congruent to n modulo n^2.

Let R_i be the product of the numbers in the i^{th} row, and C_j be the product of the number in the j^{th} column. Prove that the sums $R_1 + \ldots R_n$ and $C_1 + \ldots C_n$ are congruent modulo n^4.

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4 Find all functions $f : (0, \infty) \mapsto \mathbb{R}$ such that $(y^2 + 1)f(x) - yf(xy) = yf\left(\frac{x}{y}\right)$, for every $x, y > 0$.
