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1 Let F be the set of all $n-tuples$ $(A_1,A_2,,A_n)$ such that each A_i is a subset of $1,2,,2019$. Let $\mid A \mid$ denote the number of elements o the set A . Find

$\sum_{(A_1,,A_n)\in F} \mid A_1\cup A_2\cup ... \cup A_n \mid$

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2 Consider two circles k_1,k_2 touching at point T.
A line touches k_2 at point X and intersects k_1 at points A,B where B lies between A and X.Let S be the second intersection point of k_1 with XT. On the arc $\overset{\frown}{TS}$ not containing A and B , a point C is choosen.
Let CY be the tangent line to k_2 with $Y\in k_2$, such that the segment CY doesn't intersect the segment ST .If $I=XY\cap SC$, prove that :

(a) the points C,T,Y,I are concyclic. (b) I is the $A-excenter$ of $\triangle ABC$

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3 Find all functions $u:R\rightarrow{R}$ for which there exists a strictly monotonic function $f:R\rightarrow{R}$ such that $f(x+y)=f(x)u(y)+f(y)$
for all $x,y\in{\mathbb{R}}$

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4 Consider an odd prime number p and p consecutive positive integers $m_1,m_2,,m_p$.
Choose a permutation σ of $1,2,,p$.
Show that there exist two different numbers $k,l\in (1,2,,p)$ such that $p\mid m_k.m_{\sigma(k)}-m_l.m_{\sigma(l)}$

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5 Let x,y,z be positive real numbers such that $x^4+y^4+z^4=1$.
Determine with proof the minimum value of
$\frac{x^3}{1-x^8}+\frac{y^3}{1-y^8}+\frac{z^3}{1-z^8}$

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6 Define a sequence $a_{n n\geq 1}$ such that $a_1=1$, $a_2=2$ and a_{n+1} is the smallest positive integer m such that m hasn't yet occurred in the sequence and also $gcd(m,a_n)\neq 1$. Show that all positive integers occur in the sequence.
