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by Alireza_Amiri, MehdiGolafshan, Mr.C

– Day 1

P1 Let S is a finite set with n elements. We divided AS to m disjoint parts such that if $A, B, A \cup B$ are in the same part, then $A = B$. Find the minimum value of m .

P2 let x, y, z be positive reals , such that $x + y + z = 1399$ find the

$$\max([x]y + [y]z + [z]x)$$

($[a]$ is the biggest integer not exceeding a)

P3 let ω_1 be a circle with O_1 as its center , let ω_2 be a circle passing through O_1 with center O_2 let A be one of the intersection of ω_1 and ω_2 let x be a line tangent line to ω_1 passing from A let ω_3 be a circle passing through O_1, O_2 with its center on the line x and intersect ω_2 at P (not O_1) prove that the reflection of P through x is on ω_1

– Day 2

P4 Let ω_1 and ω_2 be two circles that intersect at point A and B . Define point X on ω_1 and point Y on ω_2 such that the line XY is tangent to both circles and is closer to B . Define points C and D the reflection of B WRT X and Y respectively. Prove that the angle $\angle CAD$ is less than 90°

P5 Call a pair of integers a and b square makers , if $ab + 1$ is a perfect square. Determine for which n is it possible to divide the set $\{1, 2, \dots, 2n\}$ into n pairs of square makers.

P6 Divide a circle into $2n$ equal sections. We call a circle *filled* if it is filled with the numbers $0, 1, 2, \dots, n - 1$. We call a filled circle *good* if it has the following properties:

i. Each number $0 \leq a \leq n - 1$ is used exactly twice *ii.* For any a we have that there are exactly a sections between the two sections that have the number a in them.

Here is an example of a good filling for $n = 5$ (View attachment)

Prove that there doesnt exist a good filling for $n = 1399$