

Turkey EGMO TST 2019

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- 1 A_1, A_2, \dots, A_n are the subsets of $|S| = 2019$ such that union of any three of them gives S but if we combine two of subsets it doesn't give us S . Find the maximum value of n .
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- 2 Let a, b, c be positive reals such that $abc = 1, a + b + c = 5$ and

$$(ab + 2a + 2b - 9)(bc + 2b + 2c - 9)(ca + 2c + 2a - 9) \geq 0$$

Find the minimum value of

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$$

- 3 Let ω be the circumcircle of $\triangle ABC$, where $|AB| = |AC|$. Let D be any point on the minor arc AC . Let E be the reflection of point B in line AD . Let F be the intersection of ω and line BE and Let K be the intersection of line AC and the tangent at F . If line AB intersects line FD at L , Show that K, L, E are collinear points
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- 4 Let $\sigma(n)$ shows the number of positive divisors of n . Let $s(n)$ be the number of positive divisors of $n + 1$ such that for every divisor $a, a - 1$ is also a divisor of n . Find the maximum value of $2s(n) - \sigma(n)$.
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- 5 Let D be the midpoint of \overline{BC} in $\triangle ABC$. Let P be any point on \overline{AD} . If the internal angle bisector of $\angle ABP$ and $\angle ACP$ intersect at Q . Prove that, if $BQ \perp QC$, then Q lies on AD
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- 6 There are k piles and there are 2019 stones totally. In every move we split a pile into two or remove one pile. Using finite moves we can reach conclusion that there are k piles left and all of them contain different number of stonws. Find the maximum of k .
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