International Mathematical Excellence Olympiad
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by MS_Kekas

– Day 1

Problem 1 Let ABC be a triangle and A' be the reflection of A about BC. Let P and Q be points on AB and AC, respectively, such that $PA' = PC$ and $QA' = QB$. Prove that the perpendicular from A' to PQ passes through the circumcenter of $\triangle ABC$.

Fedir Yudin

Problem 2 You are given an odd number $n \geq 3$. For every pair of integers (i, j) with $1 \leq i \leq j \leq n$ there is a domino, with i written on one its end and with j written on another (there are $\frac{n(n+1)}{2}$ domino overall). Amin took this dominos and started to put them in a row so that numbers on the adjacent sides of the dominos are equal. He has put k dominos in this way, got bored and went away. After this Anton came to see this k dominos, and he realized that he can't put all the remaining dominos in this row by the rules. For which smallest value of k is this possible?

Oleksii Masalitin

Problem 3 Find all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ such that for all positive real x, y holds

$$xf(x) + yf(y) = (x + y)f\left(\frac{x^2 + y^2}{x + y}\right)$$

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Fedir Yudin

– Day 2

Problem 4 Anna and Ben are playing with a permutation p of length 2020, initially $p_i = 2021 - i$ for $1 \leq i \leq 2020$. Anna has power A, and Ben has power B. Players are moving in turns, with Anna moving first.

In his turn player with power P can choose any P elements of the permutation and rearrange them in the way he/she wants.

Ben wants to sort the permutation, and Anna wants to not let this happen. Determine if Ben can make sure that the permutation will be sorted (of form $p_i = i$ for $1 \leq i \leq 2020$) in finitely many turns, if

a) $A = 1000, B = 1000$

b) $A = 1000, B = 1001$

c) $A = 1000, B = 1002$

Anton Trygub

Problem 5 For a positive integer n with prime factorization $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$ let's define $\lambda(n) = (-1)^{\alpha_1+\alpha_2+\cdots+\alpha_k}$.

Define $L(n)$ as sum of $\lambda(x)$ over all integers from 1 to n.

Define $K(n)$ as sum of $\lambda(x)$ over all **composite** integers from 1 to n.

For some $N > 1$, we know, that for every $2 \leq n \leq N$, $L(n) \leq 0$.

Prove that for this N, for every $2 \leq n \leq N$, $K(n) \geq 0$.

Mykhailo Shtandenko

Problem 6 Let O, I, and ω be the circumcenter, the incenter, and the incircle of nonequilateral $\triangle ABC$. Let ω_A be the unique circle tangent to AB and AC, such that the common chord of ω_A and ω passes through the center of ω_A . Let O_A be the center of ω_A. Define $\omega_B, O_B, \omega_C, O_C$ similarly. If ω touches BC, CA, AB at D, E, F respectively, prove that the perpendiculars from D, E, F to O_BO_C, O_CO_A, O_AO_B are concurrent on the line OI.

Pitchayut Saengrungkongka
