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Day 1 5 February 2020

1 H is the orthocenter of a non-isosceles acute triangle $\triangle ABC$. M is the midpoint of BC and BB_1, CC_1 are two altitudes of $\triangle ABC$. N is the midpoint of B_1C_1 . Prove that AH is tangent to the circumcircle of $\triangle MNH$.

2 $p(m)$ is the number of distinct prime divisors of a positive integer $m > 1$ and $f(m)$ is the $\left\lfloor \frac{p(m)+1}{2} \right\rfloor$ th smallest prime divisor of m . Find all positive integers n satisfying the equation:

$$f(n^2 + 2) + f(n^2 + 5) = 2n - 4$$

3 There are $33!$ empty boxes labeled from 1 to $33!$. In every move, we find the empty box with the smallest label, then we transfer 1 ball from every box with a smaller label and we add an additional 1 ball to that box. Find the smallest labeled non-empty box and the number of the balls in it after $33!$ moves.

Day 2 6 February 2020

4 Every square of a 2020×2020 chess table is painted in red or white. For every two columns and two rows, at least two of the intersection squares satisfies that they are in the same column or row and they are painted in the same color. Find the least value of number of columns and rows that are completely painted in one color.

5 A, B, C, D, E points are on Γ cycle clockwise. $[AE \cap [CD = \{M\}$ and $[AB \cap [DC = \{N\}$. The line parallels to EC and passes through M intersects with the line parallels to BC and passes through N on K . Similarly, the line parallels to ED and passes through M intersects with the line parallels to BD and passes through N on L . Show that the lines LD and KC intersect on Γ .

6 x, y, z are positive real numbers such that:

$$xyz + x + y + z = 6$$

$$xyz + 2xy + yz + zx + z = 10$$

Find the maximum value of:

$$(xy + 1)(yz + 1)(zx + 1)$$

