CJMO - Canadian Junior Mathematical Olympiad 2020

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1 Let $a_1, a_2, a_3, ...$ be a sequence of positive real numbers that satisfies $a_1 = 1$ and $a_{n+1}^2 + a_{n+1} = a_n$ for every natural number n. Prove that $a_n \geq \frac{1}{n}$ for every natural number n.

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2 Ziquan makes a drawing in the plane for art class. He starts by placing his pen at the origin, and draws a series of line segments, such that the n^{th} line segment has length n. He is not allowed to lift his pen, so that the end of the n^{th} segment is the start of the $(n+1)^{th}$ segment. Line segments drawn are allowed to intersect and even overlap previously drawn segments. After drawing a finite number of line segments, Ziquan stops and hands in his drawing to his art teacher. He passes the course if the drawing he hands in is an N by N square, for some positive integer N, and he fails the course otherwise. Is it possible for Ziquan to pass the course?

– those were also the first CMO problems

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3 There are $n \geq 3$ distinct positive real numbers. Show that there are at most $n-2$ different integer power of three that can be written as the sum of three distinct elements from these n numbers.

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4 $ABCD$ is a fixed rhombus. Segment PQ is tangent to the inscribed circle of $ABCD$, where P is on side AB, Q is on side AD. Show that, when segment PQ is moving, the area of ΔCPQ is a constant.

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5 There are finite many coins in Davids purse. The values of these coins are pair wisely distinct positive integers. Is that possible to make such a purse, such that David has exactly 2020 different ways to select the coins in his purse and the sum of these selected coins is 2020?
