www.artofproblemsolving.com/community/c1236109
by circlethm

– We say an integer n is naoish if $n \geq 90$ and the second-to-last digit of n (in decimal notation) is equal to 9. For example, 10798, 1999 and 90 are naoish, whereas 9900, 2009 and 9 are not. Nino expresses 2020 as a sum:

$$2020 = n_1 + n_2 + \ldots + n_k$$

where each of the n_j is naoish.

What is the smallest positive number k for which Nino can do this?

– A round table has $2N$ chairs around it. Due to social distancing guidelines, no two people are allowed to sit next to each other. How many different ways are there to choose seats around the table on which $N-1$ guests can be seated?

– Circles Ω_1, centre Q, and Ω_2, centre R, touch externally at B. A third circle, Ω_3, which contains Ω_1 and Ω_2, touches Ω_1 and Ω_2 at A and C, respectively. Point C is joined to B and the line BC is extended to meet Ω_3 at D.

Prove that QR and AD intersect on the circumference of Ω_1.

– Let n be a positive integer.
An [i]n-level honeycomb[/i] is a plane region covered with regular hexagons of side-length 1 connected along edges, such that the centres of the boundary hexagons are lined up along a regular hexagon of side-length $n\sqrt{3}$.

The diagram shows a 2-level honeycomb from which the central hexagon has been removed.

A *trex* is a sequence of 3 hexagons with collinear centres such that the middle hexagon shares an edge with each of its neighbours in the trex.

An n-level honeycomb from which the central size-1 hexagon has been removed is to be completely covered by trexes without any overlaps.

Find all values of n for which this is possible.

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– Let $a, b, c > 0$. Prove that

$$\sqrt[7]{\frac{a}{b+c}+\frac{b}{c+a}}+\sqrt[7]{\frac{b}{c+a}+\frac{c}{a+b}}+\sqrt[7]{\frac{c}{a+b}+\frac{a}{b+c}} \geq 3.$$

– Pat has a pentagon, each of whose vertices is coloured either red or blue. Once an hour, Pat recolours the vertices as follows.

- Any vertex whose two neighbors were the same colour for the last hour, becomes blue for the next hour.
- Any vertex whose two neighbors were different colours for the last hour, becomes red for the next hour.

Show that there is at least one vertex which is blue after the first recolouring and remains blue for ever.

– A function $f : \mathbb{N} \to \mathbb{N}$ satisfies the following for all $n \in \mathbb{N}$:

$$\begin{aligned} f(1) &= 1 \\ f(f(n)) &= n \\ f(2n) &= 2f(n) + 1 \end{aligned}$$

Find the value of $f(2020)$.

– Determine the last (rightmost) three decimal digits of n where:

$$n = 1 \times 3 \times 5 \times 7 \times \ldots \times 2019.$$

– A trapezium $ABCD$, in which AB is parallel to DC, is inscribed in a circle of radius R and centre O. The non-parallel sides DA and CB are extended to meet at P while diagonals AC and BD intersect at E. Prove that $|OE| \cdot |OP| = R^2$.

– Show that there exists a hexagon $ABCDEF$ in the plane such that the distance between every pair of vertices is an integer.
