Thailand Mathematical Olympiad 2014

www.artofproblemsolving.com/community/c1258388

by parmenides51, Outwitter, blackwave, reveryu

– Day 1

1 Let $\triangle ABC$ be an isosceles triangle with $\angle BAC = 100^o$. Let D, E be points on ray $\overrightarrow{AB}$ so that $BC = AD = BE$. Show that $BC \cdot DE = BD \cdot CE$

2 Find all functions $f : R \to R$ satisfying $f(xy-1) + f(x)f(y) = 2xy - 1$ for all real numbers x, y

3 Let M and N be positive integers. Pisut walks from point $(0, N)$ to point $(M, 0)$ in steps so that • each step has unit length and is parallel to either the horizontal or the vertical axis, and • each point (x, y) on the path has nonnegative coordinates, i.e. $x, y > 0.$
During each step, Pisut measures his distance from the axis parallel to the direction of his step, if after the step he ends up closer from the origin (compared to before the step) he records the distance as a positive number, else he records it as a negative number.
Prove that, after Pisut completes his walk, the sum of the signed distances Pisut measured is zero.

4 Find $P(x) \in Z[x]$ st : $P(n)|2557^n + 213.2014$ with any $n \in N^*$

– Day 2

5 Determine the maximal value of k such that the inequality

$$\left(k+\frac{a}{b}\right)\left(k+\frac{b}{c}\right)\left(k+\frac{c}{a}\right) \le \left(\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\right)\left(\frac{b}{a}+\frac{c}{b}+\frac{a}{c}\right)$$

holds for all positive reals a, b, c.

6 Find all primes p such that $2p^2 - 3p - 1$ is a positive perfect cube

7 Let $ABCD$ be a convex quadrilateral with shortest side AB and longest side CD, and suppose that $AB < CD$. Show that there is a point $E \neq C, D$ on segment CD with the following property:
For all points $P \neq E$ on side CD, if we define O_1 and O_2 to be the circumcenters of $\triangle APD$ and $\triangle BPE$ respectively, then the length of O_1O_2 does not depend on P.

8 Let n be a positive integer. We want to make up a collection of cards with the following properties:

1. each card has a number of the form $m!$ written on it, where m is a positive integer.

2. for any positive integer $t \leq n!$, we can select some card(s) from this collection such that the sum of the number(s) on the selected card(s) is t.

Determine the smallest possible number of cards needed in this collection.