Organized by Australian Maths Trust

www.artofproblemsolving.com/community/c1267963

by The_Maitreyo1

DAY 1 Determine all pairs of (a, b) of non negative integers such that:

$$\frac{a+b}{2} - \sqrt{ab} = 1$$

– Amy and Bee play the following game. Initially, there are three piles, each containing 2020 stones. The players take turns to make a move, with Amy going first. Each move consists of choosing one of the piles available, removing the unchosen pile(s) from the game, and then dividing the chosen pile into 2 or 3 non-empty piles. A player looses the game if they are unable to make a move.

Prove that Bee can always win the game,no matter how Amy plays

---

– Let ABC be a triangle with right angle at $\angle C$. Suppose that the tangent line at C to the circle passing through A, B, C intersects the line AB at D. Let E be the midpoint of CD and let F be the point on the line EB such that AF is parallel to CD. Prove that the lines AB and CF are perpendicular.

– Define the sequence $A_1, A_2, \cdots$ by $A_1 = 1$ and for $n = 2, 3, 4, \cdots$,

$$A_{n+1} = \frac{A_n + 2}{A_n + 1}$$

Define another sequence $B_1, B_2, \cdots$ by $B_1 = 1$ and for $n = 2, 3, 4, \cdots$,

$$B_{n+1} = \frac{B_n^2 + 2}{2B_n}$$

Prove that $B_{n+1} = A_{2^n}$ for all non-negative integers n .

DAY 2 Each term of an infinite sequene $a_1, a_2, \cdots$ is equal to 0 or 1. For each positive integer n,

- $a_n + a_{n+1} \neq a_{n+2} + a_{n+3}$ and
- $a_n + a_{n+1} + a_{n+2} \neq a_{n+3} + a_{n+4} + a_{n+5}$

Prove that if $a_1 = 0$, then $a_{2020} = 1$

– Upvote if you got IMO 2011 vibes :rotfl:

Let $ABCD$ be a square. For a point P inside it, a *windmill* centred at P consists of 2 perpendicular lines l_1 and l_2 passing through P , such that • l_1 intersects the sides AB and CD at W and Y, respectively, and • l_2 intersects the side BC and DA at X and Z, respectively.

A windmill is called *round* if the quadrilateral WXYZ is **cyclic.** Determine all points P inside the square such that every windmill centred at P is round.

– A *tetramino* tile is a tile that can be formed by gluing together four unit square tiles, edge to edge. For each natural number n, consider a bathroom whose floor is in the shape of a $2 * 2n$ rectangle. Let T_n be the number of ways to tile this bathroom floor with tetramino tiles. Prove that, each of the numbers $T_1, T_2, T_3, \cdots$ is a perfect square.

– Prove that for each integer K satisfying $2 \leq k \leq 100$, there are positive integers $b_2, b_3, \cdots, b_{101}$ such that:

$$b_2^2 + b_3^3 + \cdots + b_k^k \ = \ b_k^k + \cdots + b_{101}^{101}$$

@below, typo rectified!
