The problems from the 24th Junior Macedonian Mathematical Olympiad

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1 Let S be the set of all positive integers n such that each of the numbers $n+1$, $n+3$, $n+4$, $n+5$, $n+6$, and $n+8$ is composite. Determine the largest integer k with the following property: For each $n \in S$ there exist at least k consecutive composite integers in the set $n, n+1, n+2, n+3, n+4, n+5, n+6, n+7, n+8, n+9$.

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2 Let $x, y,$ and z be positive real numbers such that $xy + yz + zx = 27$. Prove that

$$x + y + z \geq \sqrt{3xyz}.$$

When does equality hold?

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3 Solve the following equation in the set of integers

$$x^5 + 2 = 3 \cdot 101^y.$$

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4 Let ABC be an isosceles triangle with base AC. Points D and E are chosen on the sides AC and BC, respectively, such that $CD = DE$. Let $H, J,$ and K be the midpoints of $DE, AE,$ and BD, respectively. The circumcircle of triangle DHK intersects AD at point F, whereas the circumcircle of triangle HEJ intersects BE at G. The line through K parallel to AC intersects AB at I. Let $IH \cap GF = M$. Prove that $J, M,$ and K are collinear points.

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5 Let T be a triangle whose vertices have integer coordinates, such that each side of T contains exactly m points with integer coordinates. If the area of T is less than 2020, determine the largest possible value of m.
