Mediterranean Mathematics Olympiad 2020

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by parmenides51

1 Determine all integers $m \geq 2$ for which there exists an integer $n \geq 1$ with $\gcd(m,n) = d$ and $\gcd(m, 4n+1) = 1$.

Proposed by Gerhard Woeginger, Austria

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2 Let S be a set of $n \geq 2$ positive integers. Prove that there exist at least n^2 integers that can be written in the form $x + yz$ with $x, y, z \in S$.

Proposed by Gerhard Woeginger, Austria

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3 Prove that all postive real numbers a, b, c with $a+b+c=4$ satisfy the inequality

$$\frac{ab}{\sqrt[4]{3c^2+16}} + \frac{bc}{\sqrt[4]{3a^2+16}} + \frac{ca}{\sqrt[4]{3b^2+16}} \leq \frac{4}{3}\sqrt[4]{12}$$

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4 Let P, Q, R be three points on a circle k_1 with $|PQ| = |PR|$ and $|PQ| > |QR|$. Let k_2 be the circle with center in P that goes through Q and R. The circle with center Q through R intersects k_1 in another point $X \neq R$ and intersects k_2 in another point $Y \neq R$. The two points X and R lie on different sides of the line through PQ. Show that the three points P, X, Y lie on a common line.
