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by Tintarn, Kezer, hajimbrak

– VAIMO 1

1 Find the least positive integer n, such that there is a polynomial

$$P(x) = a_{2n}x^{2n} + a_{2n-1}x^{2n-1} + \dots + a_1x + a_0$$

with real coefficients that satisfies both of the following properties:
- For $i = 0, 1, \dots, 2n$ it is $2014 \leq a_i \leq 2015$.
- There is a real number ξ with $P(\xi) = 0$.

2 A positive integer n is called *naughty* if it can be written in the form $n = a^b + b$ with integers $a, b \geq 2.$
Is there a sequence of 102 consecutive positive integers such that exactly 100 of those numbers are naughty?

3 Let ABC be an acute triangle with $|AB| \neq |AC|$ and the midpoints of segments $[AB]$ and $[AC]$ be D resp. E. The circumcircles of the triangles BCD and BCE intersect the circumcircle of triangle ADE in P resp. Q with $P \neq D$ and $Q \neq E$.
Prove $|AP| = |AQ|$.

[i](Notation: $|\cdot|$ denotes the length of a segment and $[\cdot]$ denotes the line segment.)[/i]

– VAIMO 2

1 Determine all pairs (x, y) of positive integers such that

$$\sqrt[3]{7x^2 - 13xy + 7y^2} = |x - y| + 1.$$

Proposed by Titu Andreescu, USA

2 Let ABC be an acute triangle with the circumcircle k and incenter I. The perpendicular through I in CI intersects segment $[BC]$ in U and k in V. In particular V and A are on different sides of BC. The parallel line through U to AI intersects AV in X.
Prove: If XI and AI are perpendicular to each other, then XI intersects segment $[AC]$ in its midpoint M.

[i](Notation: $[\cdot]$ denotes the line segment.)[/i]

3 Construct a tetromino by attaching two 2×1 dominoes along their longer sides such that the midpoint of the longer side of one domino is a corner of the other domino. This construction yields two kinds of tetrominoes with opposite orientations. Let us call them S- and Z-tetrominoes, respectively.
Assume that a lattice polygon P can be tiled with S-tetrominoes. Prove that no matter how we tile P using only S- and Z-tetrominoes, we always use an even number of Z-tetrominoes.

Proposed by Tamas Fleiner and Peter Pal Pach, Hungary
