

**NMO 2014**

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– Day 1

**1** The ship *Meridiano do Bacalhau* does its fishing business during 64 days. Each day the captain chooses a direction which may be either north or south and the ship sails that direction in that day. On the first day of business the ship sails 1 mile, on the second day sails 2 miles; generally, on the  $n$ -th day it sails  $n$  miles. After of the 64-th day, the ship was 2014 miles north from its initial position. What is the greatest number of days that the ship could have sailed south?

**2** Let  $[ABCD]$  be a square,  $M$  a point on the segment  $[AD]$ , and  $N$  a point on the segment  $[DC]$  such that  $\hat{BMA} = \hat{NMD} = 60^\circ$ . Calculate  $M\hat{B}N$ .

**3** Amlia and Beatriz play battleship on a  $2n \times 2n$  board, using very peculiar rules. Amlia begins by choosing  $n$  lines and  $n$  columns of the board, placing her  $n^2$  submarines on the cells that lie on their intersections. Next, Beatriz chooses a set of cells that will explode. Which is the least number of cells that Beatriz has to choose in order to assure that at least a submarine will explode?

– Day 2

**4** Determine all natural numbers  $x, y$  and  $z$ , such that  $x \leq y \leq z$  and

$$\left(1 + \frac{1}{x}\right) \left(1 + \frac{1}{y}\right) \left(1 + \frac{1}{z}\right) = 3.$$

**5** Let  $[ABCD]$  be a convex quadrilateral with area 2014, and let  $P$  be a point on  $[AB]$  and  $Q$  a point on  $[AD]$  such that triangles  $[ABQ]$  and  $[ADP]$  have area 1. Let  $R$  be the intersection of  $[AC]$  and  $[PQ]$ . Determine  $\frac{RC}{RA}$ .

**6** One hundred musicians are planning to organize a festival with several concerts. In each concert, while some of the one hundred musicians play on stage, the others remain in the audience assisting to the players. What is the least number of concerts so that each of the musicians has the chance to listen to each and every one of the other musicians on stage?