NMO 2013

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–	Day 1
1	Consider a parallelogram $[ABCD]$ such that $\angle DAB$ is an acute angle. Let G be a point in line AB different from B such that $\overline{BC} = \overline{GC}$, and let H be a point in line BC different from B such that $\overline{AB} = \overline{AH}$. Prove that triangle $[GDH]$ is isosceles.
2	In the morning, three people, A, B and C run in a same line at a beach in Albufeira. Some day, the three people were in the same point of the beach and then they started to run at the same time, but in different velocities. For each person, the velocity was constant. When someone arrived in an extreme of the beach, he/she turned back and runned in the opposite direction. In the moment in that the three people were in the same point of the beach again, the running finished. Not counting with the beginning and the final of the running, A met B six times and A met C eight times. How many times did B and C meet?
3	In the Republic of Unistan there are n national roads, each of them links two cities exactly. You can travel from one city to another of your choice using a sequence of roads. The President of Unistan ordered to label the national roads with the integers from 1 to n by an old law: if a city is adjacent to more than one road, the greatest common divisor of the numbers of that roads must be one. Show that you can label the national roads without breaking the law.
–	Day 2
4	Which is the leastest natural number n such that $n!$ has, at least, 2013 divisors?
5	Liliana wants to paint a $m \times n$ board. Liliana divides each unit square by one of its diagonals and paint one of the halves of the square with black and the other half with white in such a way that triangles that have a common side haven't the same colour. How many possibilities has Liliana to paint the board?
6	In each side of a regular polygon with n sides, we choose a point different from the vertices and we obtain a new polygon of n sides. For which values of n can we obtain a polygon such that the internal angles are all equal but the polygon isn't regular?