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Lusophon Mathematical Olympiad 2020

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–	Day 1
1	In certain country, the coins have the following values: $2^0, 2^1, 2^2, \dots 2^{10}$. A cash machine has 1000 coins of each value and give the money using each coin(of each value) at most once. The customers order all the positive integers: $1, 2, 3, 4, 5, \dots$ (in this order) in coins. a) Determine the first integer, such that the cash machine cannot provide. b) In the moment that the first customer can not be attended, by the lack of coins, what are the coins which are not available in the cash machine?
2	a) Find a pair(s) of integers (x, y) such that: $y^2 = x^3 + 2017$ b) Prove that there isn't integers x and y, with y not divisible by 3, such that: $y^2 = x^3 - 2017$
3	Let ABC be a triangle and on the sides we draw, externally, the squares $BADE, CBFG$ and $ACHI$. Determine the greatest positive real constant k such that, for any triangle $\triangle ABC$, the following inequality is true: $[DEFGHI] \geq k \cdot [ABC]$ Note: $[X]$ denotes the area of polygon X.
–	Day 2
4	Let ABC be an acute triangle. Its incircle touches the sides BC, CA and AB at the points D, E and F, respectively. Let P, Q and R be the circumcenters of triangles AEF, BDF and CDE, respectively. Prove that triangles ABC and PQR are similar.
5	In how many ways can we fill the cells of a 4×4 grid such that each cell contains exactly one positive integer and the product of the numbers in each row and each column is 2020?
6	Prove that $\lfloor \sqrt{9n+7} \rfloor = \lfloor \sqrt{n} + \sqrt{n+1} + \sqrt{n+2} \rfloor$ for all postive integer n.