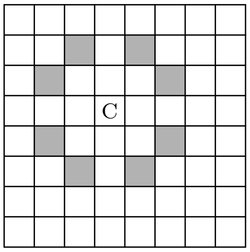
Nivel Mayor , L2, final round of 2003 Chile NMO

www.artofproblemsolving.com/community/c1459075

by parmenides51

1 Investigate whether a chess knight can traverse a 4×4 mini-chessboard so that it reaches each of the 16 squares only once.

Note: the drawing below shows the endpoints of the eight possible moves of the knight (C) on a chessboard of size 8×8.

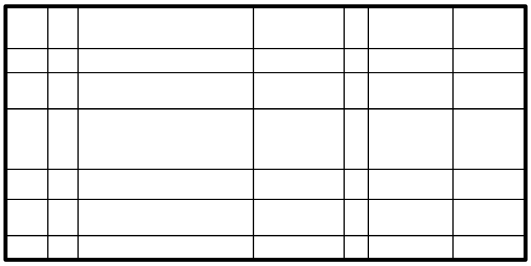


---

2 Find all primes p, q such that $p + q = (p - q)^3$.

---

3 A rectangle is decomposed by 6 vertical lines and 6 horizontal lines in the 49 small rectangles (see figure). The perimeter of each small rectangle is known to be a whole number of meters. In this case, will the perimeter of the large rectangle be a whole number of meters?



---

4 Juan did not like the criticism of his classmates published in his school newspaper. He found nothing better than to start ripping up the diary. First he tore it into 4 parts and then he continued to break it in a very methodical way: namely, each piece of newspaper he found he would tear it back into 4 or 10 pieces randomly. Breaking this way, was he able to get exactly 2003 pieces of the diary?

---

5 Prove that there is a natural number N of the form $11...1100...00$ which is divisible by 2003. (The natural numbers are: $1, 2, 3, ...$)

---

6 Consider a triangle ABC. On the line AC take a point B_1 such that $AB = AB_1$ and in addition, B_1 and C are located on the same side of the line with respect to the point A. The bisector of the angle A intersects the side BC at a point that we will denote as A_1. Let P and R be the circumscribed circles of the triangles ABC and A_1B_1C respectively. They intersect at points C and Q. Prove that the tangent to the circle R at the point Q is parallel to the line AC.

---

7 Juan found an easy (but wrong) way to simplify fractions. He proposes to simplify a fraction $\frac{M}{N}$, where $M < N$ are two natural numbers, erase simultaneously the equal digits in the numerator and denominator. For instance, $\frac{12356}{5789}$ transforms after simplification of Juan in $\frac{126}{789}$. Find out if there is at least one fraction $\frac{M}{N}$, with $10 < M < N < 100$ for which this method gives a correct result.
