Kazakhstan National Olympiad 2006

www.artofproblemsolving.com/community/c1459269

by parmenides51

–	grade 11
–	day 1
1	Natural numbers from 1 to 200 were divided into 50 sets. Prove that one of them contains three numbers that are the lengths of the sides of some triangle
2	Product of square trinomials $x^2+a_1x+b_1$, $x^2+a_2x+b_2$, $\dots$, $x^2+a_nx+b_n$ equals polynomial $P(x)=x^{2n}+c_1x^{2n-1}+c_2x^{2n-2}+\dots+c_{2n-1}x+c_{2n}$, where the coefficients c_1, c_2, $\dots$, c_{2n} are positive. Prove that for some k ($1\leq k\leq n$) the coefficients a_k and b_k are positive.
3	The racing tournament has 12 stages and n participants. After each stage, all participants, depending on the occupied place k, receive points a_k (the numbers a_k are natural and $a_1>a_2>\dots>a_n$). For what is the smallest n the tournament organizer can choose the numbers a_1, $\dots$, a_n so that after the penultimate stage for any possible distribution of places at least two participants had a chance to take first place.
4	grade IX P4, X P3 The bisectors of the angles A and C of the triangle ABC intersect the circumscribed circle of this triangle at the points A_0 and C_0, respectively. The straight line passing through the center of the inscribed circle of triangle ABC parallel to the side of AC, intersects with the line A_0C_0 at P. Prove that the line PB is tangent to the circumcircle of the triangle ABC. grade XI P4 The bisectors of the angles A and C of the triangle ABC intersect the sides at the points A_1 and C_1, and the circumcircle of this triangle at points A_0 and C_0 respectively. Straight lines A_1C_1 and A_0C_0 intersect at point P. Prove that the segment connecting P with the center inscribed circles of triangle ABC, parallel to AC.
–	day 2
5	Prove that for every x such that $\sin x\neq 0$, exists natural n such that $\lvert\sin nx\rvert\geq\frac{\sqrt{3}}{2}$.
6	In the tetrahedron $ABCD$ from the vertex A, the perpendiculars AB', AC' are drawn, AD' on planes dividing dihedral angles at edges CD, BD, BC in half. Prove that the plane $(B'C'D')$ is parallel to the plane (BCD).

7 Prove that if a natural number N can be represented in the form the sum of three squares of integers divisible by 3, then it is also is represented as the sum of three squares of integers that are not divisible by 3.

---

8 What is the minimum number of cells that can be colored black in white square 300×300 so that no three black cells formed a corner, and after painting any white cell this condition violated?
