Kazakhstan National Olympiad 2016

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–	grade 11
–	day 1
1	Prove that one can arrange all positive divisors of any given positive integer around a circle so that for any two neighboring numbers one is divisible by another.
2	Find all rational numbers a,for which there exist infinitely many positive rational numbers q such that the equation $[x^a].x^a = q$ has no solution in rational numbers.(A.Vasiliev)
3	Circles ω_1, ω_2 intersect at points X, Y and they are internally tangent to circle Ω at points A, B,respectively.AB intersect with ω_1, ω_2 at points A_1, B_1 ,respectively.Another circle is internally tangent to ω_1, ω_2 and A_1B_1 at Z.Prove that $\angle AXZ = \angle BXZ$.(C.Ilyasov)
–	day 2
4	In isosceles triangle $ABC(CA = CB)$,CH is altitude and M is midpoint of BH.Let K be the foot of the perpendicular from H to AC and $L = BK \cap CM$ .Let the perpendicular drawn from B to BC intersects with HL at N.Prove that $\angle ACB = 2\angle BCN$.(M. Kunhozhyn)
5	101 blue and 101 red points are selected on the plane, and no three lie on one straight line. The sum of the pairwise distances between the red points is 1 (that is, the sum of the lengths of the segments with ends at red points), the sum of the pairwise distances between the blue ones is also 1, and the sum of the lengths of the segments with the ends of different colors is 400. Prove that you can draw a straight line separating everything red dots from all blue ones.
6	Given a strictly increasing infinite sequence $\{a_n\}$ of positive real numbers such that for any $n \in N$: $$a_{n+2} = (a_{n+1} - a_n)^{\sqrt{n}} + n^{-\sqrt{n}}$$ Prove that for any $C > 0$ there exist a positive integer $m(C)$ (depended on C) such that $a_{m(C)} > C$.