7th IGO

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–	Elementary
1	By a *fold* of a polygon-shaped paper, we mean drawing a segment on the paper and folding the paper along that. Suppose that a paper with the following figure is given. We cut the paper along the boundary of the shaded region to get a polygon-shaped paper. Start with this shaded polygon and make a rectangle-shaped paper from it with at most 5 number of folds. Describe your solution by introducing the folding lines and drawing the shape after each fold on your solution sheet. (Note that the folding lines do not have to coincide with the grid lines of the shape.) *Proposed by Mahdi Etesamifard*
2	A parallelogram $ABCD$ is given ($AB \neq BC$). Points E and G are chosen on the line $\overline{CD}$ such that $\overline{AC}$ is the angle bisector of both angles $\angle EAD$ and $\angle BAG$. The line $\overline{BC}$ intersects $\overline{AE}$ and $\overline{AG}$ at F and H, respectively. Prove that the line $\overline{FG}$ passes through the midpoint of HE. *Proposed by Mahdi Etesamifard*
3	According to the figure, three equilateral triangles with side lengths a, b, c have one common vertex and do not have any other common point. The lengths x, y, and z are defined as in the figure. Prove that $3(x+y+z) > 2(a+b+c)$. *Proposed by Mahdi Etesamifard*
4	Let P be an arbitrary point in the interior of triangle $\triangle ABC$. Lines$\overline{BP}$ and $\overline{CP}$ intersect $\overline{AC}$ and $\overline{AB}$ at E and F, respectively. Let K and L be the midpoints of the segments BF and CE, respectively. Let the lines through L and K parallel to $\overline{CF}$ and $\overline{BE}$ intersect $\overline{BC}$ at S and T, respectively; moreover, denote by M and N the reflection of S and T over the points L and K, respectively. Prove that as P moves in the interior of triangle $\triangle ABC$, line $\overline{MN}$ passes through a fixed point. *Proposed by Ali Zamani*
5	We say two vertices of a simple polygon are *visible* from each other if either they are adjacent, or the segment joining them is completely inside the polygon (except two endpoints that lie on the boundary). Find all positive integers n such that there exists a simple polygon with n vertices in which every vertex is visible from exactly 4 other vertices. (A simple polygon is a polygon without hole that does not intersect itself.) *Proposed by Morteza Saghafian*

–	Intermediate
1	A trapezoid $ABCD$ is given where AB and CD are parallel. Let M be the midpoint of the segment AB. Point N is located on the segment CD such that $\angle ADN = \frac{1}{2}\angle MNC$ and $\angle BCN = \frac{1}{2}\angle MND$. Prove that N is the midpoint of the segment CD. *Proposed by Alireza Dadgarnia*
2	Let ABC be an isosceles triangle ($AB = AC$) with its circumcenter O. Point N is the midpoint of the segment BC and point M is the reflection of the point N with respect to the side AC. Suppose that T is a point so that $ANBT$ is a rectangle. Prove that $\angle OMT = \frac{1}{2}\angle BAC$. *Proposed by Ali Zamani*
3	In acute-angled triangle ABC ($AC > AB$), point H is the orthocenter and point M is the midpoint of the segment BC. The median AM intersects the circumcircle of triangle ABC at X. The line CH intersects the perpendicular bisector of BC at E and the circumcircle of the triangle ABC again at F. Point J lies on circle ω, passing through $X, E,$ and F, such that $BCHJ$ is a trapezoid ($CB \parallel HJ$). Prove that JB and EM meet on ω. *Proposed by Alireza Dadgarnia*
4	Triangle ABC is given. An arbitrary circle with center J, passing through B and C, intersects the sides AC and AB at E and F, respectively. Let X be a point such that triangle FXB is similar to triangle EJC (with the same order) and the points X and C lie on the same side of the line AB. Similarly, let Y be a point such that triangle EYC is similar to triangle FJB (with the same order) and the points Y and B lie on the same side of the line AC. Prove that the line XY passes through the orthocenter of the triangle ABC. *Proposed by Nguyen Van Linh - Vietnam*
5	Find all numbers $n \geq 4$ such that there exists a convex polyhedron with exactly n faces, whose all faces are right-angled triangles. (Note that the angle between any pair of adjacent faces in a convex polyhedron is less than 180°.) *Proposed by Hesam Rajabzadeh*
–	Advanced
1	Let M, N, P be midpoints of BC, AC and AB of triangle $\triangle ABC$ respectively. E and F are two points on the segment $\overline{BC}$ so that $\angle NEC = \frac{1}{2}\angle AMB$ and $\angle PFB = \frac{1}{2}\angle AMC$. Prove that

$AE = AF$.
Proposed by Alireza Dadgarnia

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2 Let $\triangle ABC$ be an acute-angled triangle with its incenter I. Suppose that N is the midpoint of the arc BAC of the circumcircle of triangle $\triangle ABC$, and P is a point such that $ABPC$ is a parallelogram.Let Q be the reflection of A over N and R the projection of A on $\overline{QI}$. Show that the line $\overline{AI}$ is tangent to the circumcircle of triangle $\triangle PQR$
Proposed by Patrik Bak - Slovakia

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3 Assume three circles mutually outside each other with the property that every line separating two of them have intersection with the interior of the third one. Prove that the sum of pairwise distances between their centers is at most $2\sqrt{2}$ times the sum of their radii.
(A line separates two circles, whenever the circles do not have intersection with the line and are on different sides of it.)
Note. Weaker results with $2\sqrt{2}$ replaced by some other c may be awarded points depending on the value of $c > 2\sqrt{2}$
Proposed by Morteza Saghafian

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4 Convex circumscribed quadrilateral $ABCD$ with its incenter I is given such that its incircle is tangent to $\overline{AD}, \overline{DC}, \overline{CB}$, and $\overline{BA}$ at K, L, M, and N. Lines $\overline{AD}$ and $\overline{BC}$ meet at E and lines $\overline{AB}$ and $\overline{CD}$ meet at F. Let $\overline{KM}$ intersects $\overline{AB}$ and $\overline{CD}$ at X, Y, respectively. Let $\overline{LN}$ intersects $\overline{AD}$ and $\overline{BC}$ at Z, T, respectively. Prove that the circumcircle of triangle $\triangle XFY$ and the circle with diameter EI are tangent if and only if the circumcircle of triangle $\triangle TEZ$ and the circle with diameter FI are tangent.
Proposed by Mahdi Etesamifard

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5 Consider an acute-angled triangle $\triangle ABC$ ($AC > AB$) with its orthocenter H and circumcircle Γ.Points M,P are midpoints of BC and AH respectively.The line $\overline{AM}$ meets Γ again at X and point N lies on the line $\overline{BC}$ so that $\overline{NX}$ is tangent to Γ.
Points J and K lie on the circle with diameter MP such that $\angle AJP = \angle HNM$ (B and J lie one the same side of $\overline{AH}$) and circle ω_1, passing through K, H, and J, and circle ω_2 passing through K, M, and N, are externally tangent to each other. Prove that the common external tangents of ω_1 and ω_2 meet on the line $\overline{NH}$.
Proposed by Alireza Dadgarnia
