

The CNCM Online Round 3 contest was a live-scoring contest held on November 7, 2020.

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by wolfpack

- 1 Harry, who is incredibly intellectual, needs to eat carrots C_1, C_2, C_3 and solve *Daily Challenge* problems D_1, D_2, D_3 . However, he insists that carrot C_i must be eaten only after solving *Daily Challenge* problem D_i . In how many satisfactory orders can he complete all six actions?

Proposed by Albert Wang (awang2004)

- 2 Consider rectangle $ABCD$ with $AB = 6$ and $BC = 8$. Pick points E, F, G, H such that the angles $\angle AEB, \angle BFC, \angle CGD, \angle AHD$ are all right. What is the largest possible area of quadrilateral $EFGH$?

Proposed by Akshar Yeccherla (TopNotchMath)

- 3 Let $a_1 = 1$ and $a_{n+1} = a_n \cdot p_n$ for $n \geq 1$ where p_n is the n th prime number, starting with $p_1 = 2$. Let $\tau(x)$ be equal to the number of divisors of x . Find the remainder when

$$\sum_{n=1}^{2020} \sum_{d|a_n} \tau(d)$$

is divided by 91 for positive integers d . Recall that $d|a_n$ denotes that d divides a_n .

Proposed by Minseok Eli Park (wolfpack)

- 4 Hari is obsessed with cubics. He comes up with a cubic with leading coefficient 1, rational coefficients and real roots $0 < a < b < c < 1$. He knows the following three facts: $P(0) = -\frac{1}{8}$, the roots form a geometric progression in the order a, b, c , and

$$\sum_{k=1}^{\infty} (a^k + b^k + c^k) = \frac{9}{2}$$

The value $a + b + c$ can be expressed as $\frac{m}{n}$, where m, n are relatively prime positive integers. Find $m + n$.

Proposed by Akshar Yeccherla (TopNotchMath)

- 5 How many positive integers N less than 10^{1000} are such that N has x digits when written in base ten and $\frac{1}{N}$ has x digits after the decimal point when written in base ten? For example, 20 has two digits and $\frac{1}{20} = 0.05$ has two digits after the decimal point, so 20 is a valid N .

Proposed by Hari Desikan (HariDesikan)

- 6 Triangle ABC has side lengths $AB = 13$, $BC = 14$, and $CA = 15$. Let Γ denote the circumcircle of $\triangle ABC$. Let H be the orthocenter of $\triangle ABC$. Let AH intersect Γ at a point D other than A . Let BH intersect AC at F and Γ at point G other than B . Suppose DG intersects AC at X . Compute the greatest integer less than or equal to the area of quadrilateral $HDXF$.

Proposed by Kenan Hasanaliyev (claserken)

- 7 A subset of the positive integers S is said to be a *configuration* if $200 \notin S$ and for all nonnegative integers x , $x \in S$ if and only if both $2x \in S$ and $\lfloor \frac{x}{2} \rfloor \in S$. Let the number of subsets of $\{1, 2, 3, \dots, 130\}$ that are equal to the intersection of $\{1, 2, 3, \dots, 130\}$ with some configuration S equal k . Compute the remainder when k is divided by 1810.

Proposed Hari Desikan (HariDesikan)