Greece JBMO TST 2020

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1 Let ABC be a triangle with $AB > AC$. Let D be a point on side AB such that $BD = AC$. Consider the circle γ passing through point D and tangent to side AC at point A. Consider the circumscribed circle ω of the triangle ABC that interesects the circle γ at points A and E. Prove that point E is the intersection point of the perpendicular bisectors of line segments BC and AD.

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2 Let a, b, c be positive real numbers such that $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=3$. Prove that

$$\frac{a+b}{a^2+ab+b^2}+\frac{b+c}{b^2+bc+c^2}+\frac{c+a}{c^2+ca+a^2}\leq 2$$

When is the equality valid?

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3 Find all pairs (a, b) of prime positive integers a, b such that number $A = 3a^2b + 16ab^2$ equals to a square of an integer.

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4 Let A and B be two non-empty subsets of $X = \{1, 2, ..., 8\}$ with $A \cup B = X$ and $A \cap B = \emptyset$. Let P_A be the product of all elements of A and let P_B be the product of all elements of B. Find the minimum possible value of sum $P_A + P_B$.

PS. It is a variation of JBMO Shortlist 2019 A3 (`https://artofproblemsolving.com/community/c6h2267998p17621980`)
