2020 HKIMO Preliminary Selection Contest

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1 Let $n=(10^{2020}+2020)^2$. Find the sum of all the digits of n.

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2 Let x, y, z be positive integers satisfying $x<y<z$ and $x+xy+xyz=37$. Find the greatest possible value of $x+y+z$.

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3 A child lines up 2020^2 pieces of bricks in a row, and then remove bricks whose positions are square numbers (i.e. the 1st, 4th, 9th, 16th, ... bricks). Then he lines up the remaining bricks again and remove those that are in a 'square position'. This process is repeated until the number of bricks remaining drops below 250. How many bricks remain in the end?

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4 In a game, a participant chooses a nine-digit positive integer $\overline{ABCDEFGHI}$ with distinct non-zero digits. The score of the participant is $A^{B^{C^{D^{E^{F^{G^{H^{I}}}}}}}}$. Which nine-digit number should be chosen in order to maximise the score?

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5 The 28 students of a class are seated in a circle. They then all claim that 'the two students next to me are of different genders'. It is known that all boys are lying while exactly 3 girls are lying. How many girls are there in the class?

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6 In ΔABC, $AB=6$, $BC=7$ and $CA=8$. Let D be the mid-point of minor arc AB on the circumcircle of ΔABC. Find AD^2

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7 Solve the equation $\sqrt{7-x}=7-x^2$, where $x>0$.

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8 Find the smallest positive multiple of 77 whose last four digits (from left to right) are 2020.

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9 In ΔABC, $\angle B=46.6^\circ$. D is a point on BC such that $\angle BAD=20.1^\circ$. If $AB=CD$ and $\angle CAD=x^\circ$, find x.

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10 Let k be an integer. If the equation $(x-1)|x+1|=x+\frac{k}{2020}$ has three distinct real roots, how many different possible values of k are there?

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11 Let a, b, c be the three roots of the equation $x^3-(k+1)x^2+kx+12=0$, where k is a real number. If $(a-2)^3+(b-2)^3+(c-2)^3=-18$, find the value of k.

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12 There are some balls, on each of which a positive integer not exceeding 14 (and not necessarily distinct) is written, and the sum of the numbers on all balls is S. Find the greatest possible

value of S such that, regardless of what the integers are, one can ensure that the balls can be divided into two piles so that the sum of the numbers on the balls in each pile does not exceed 129.

13 There are n different integers on the blackboard. Whenever two of these integers are chosen, either their sum or difference (possibly both) will be a positive integral power of 2. Find the greatest possible value of n.

14 In ΔABC, $\angle ABC = 120^\circ$. The internal bisector of $\angle B$ meets AC at D. If $BD = 1$, find the smallest possible value of $4BC + AB$.

15 How many ten-digit positive integers consist of ten different digits and are divisible by 99?

16 ΔABC is right-angled at B, with $AB = 1$ and $BC = 3$. E is the foot of perpendicular from B to AC. BA and BE are produced to D and F respectively such that D, F, C are collinear and $\angle DAF = \angle BAC$. Find the length of AD.

17 How many positive integer solutions does the following system of equations have?

$$\begin{cases} \sqrt{2020}(\sqrt{a}+\sqrt{b}) = \sqrt{(c+2020)(d+2020)} \\ \sqrt{2020}(\sqrt{b}+\sqrt{c}) = \sqrt{(d+2020)(a+2020)} \\ \sqrt{2020}(\sqrt{c}+\sqrt{d}) = \sqrt{(a+2020)(b+2020)} \\ \sqrt{2020}(\sqrt{d}+\sqrt{a}) = \sqrt{(b+2020)(c+2020)} \end{cases}$$

18 Two n-sided polygons are said to be of the same type if we can label their vertices in clockwise order as A_1, A_2, ..., A_n and B_1, B_2, ..., B_n respectively such that each pair of interior angles A_i and B_i are either both reflex angles or both non-reflex angles. How many different types of 11-sided polygons are there?

19 Four couples are to be seated in a row. If it is required that each woman may only sit next to her husband or another woman, how many different possible seating arrangements are there?

20 Consider the Fibonacci sequence $1, 1, 2, 3, 5, 8, 13$, ... What are the last three digits (from left to right) of the 2020th term?