Bundeswettbewerb Mathematik 2020

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–	Round 1
1	Show that there are infinitely many perfect squares of the form $50^m - 50^n$, but no perfect square of the form $2020^m + 2020^n$, where m and n are positive integers.
2	Konstantin moves a knight on a $n \times n$- chess board from the lower left corner to the lower right corner with the minimal number of moves. Then Isabelle takes the knight and moves it from the lower left corner to the upper right corner with the minimal number of moves. For which values of n do they need the same number of moves?
3	Let AB be the diameter of a circle k and let E be a point in the interior of k. The line AE intersects k a second time in $C \neq A$ and the line BE intersects k a second time in $D \neq B$. Show that the value of $AC \cdot AE + BD \cdot BE$ is independent of the choice of E.
4	Define a sequence (a_n) recursively by $a_1 = 0, a_2 = 2, a_3 = 3$ and $a_n = \max_{0<d<n} a_d \cdot a_{n-d}$ for $n \geq 4$. Determine the prime factorization of $a_{19702020}$.
–	Round 2
1	Leo and Smilla find 2020 gold nuggets with masses $1, 2, \dots, 2020$ gram, which they distribute to a red and a blue treasure chest according to the following rule: First, Leo chooses one of the chests and tells its colour to Smilla. Then Smilla chooses one of the not yet distributed nuggets and puts it into this chest. This is repeated until all the nuggets are distributed. Finally, Smilla chooses one of the chests and wins all the nuggets from this chest. How many gram of gold can Smilla make sure to win?
2	Prove that there are no rational numbers x, y, z with $x + y + z = 0$ and $x^2 + y^2 + z^2 = 100$.
3	Two lines m and n intersect in a unique point P. A point M moves along m with constant speed, while another point N moves along n with the same speed. They both pass through the point P, but not at the same time.

Show that there is a fixed point $Q \neq P$ such that the points P, Q, M and N lie on a common circle all the time.

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4 In each cell of a table with m rows and n columns, where $m < n$, we put a non-negative real number such that each column contains at least one positive number.

Show that there is a cell with a positive number such that the sum of the numbers in its row is larger than the sum of the numbers in its column.
