Austrian Regional Competition For Advanced Students 2020

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1 Let a be a positive integer. Determine all a such that the equation

$$\left(1+\frac{1}{x}\right)\cdot\left(1+\frac{1}{x+1}\right)\cdots\left(1+\frac{1}{x+a}\right)=a-x$$

has at least one integer solution for x.
For every such a state the respective solutions.

(Richard Henner)

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2 The set M consists of all 7-digit positive integer numbers that contain (in decimal notation) each of the digits $1, 3, 4, 6, 7, 8$ and 9 exactly once.
(a) Find the smallest positive difference d of two numbers from M.
(b) How many pairs (x, y) with x and y from M are there for which $x - y = d$?

(Gerhard Kirchner)

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3 Let a triangle ABC be given with $AB < AC$. Let the inscribed center of the triangle be I. The perpendicular bisector of side BC intersects the angle bisector of BAC at point S and the angle bisector of CBA at point T. Prove that the points C, I, S and T lie on a circle.

(Karl Czakler)

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4 Find all quadruples (p, q, r, n) of prime numbers p, q, r and positive integer numbers n, such that

$$p^2 = q^2 + r^n$$

(Walther Janous)
